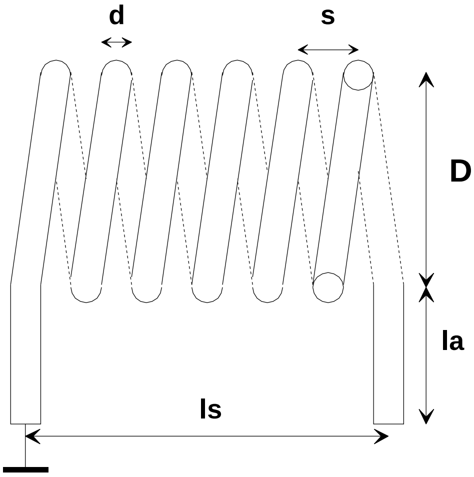
Calculation of the Single Layer Solenoid

SI - System



- Volume of coil V [m^3]
- Diameter of coil D [m]
- Length of coil ls [m]
- Formfactor ls/D α [-]
- Number of turns n [-]
- Diameter of wire d [m]
- Pitch s [m]
- Copper density d/s η [-]
- Operating frequency f [Hz]
- Skin- depth δ [m]]
- Length of connection la[m]

further definitions:

$\alpha = \frac{ls}{D}$ $\eta = \frac{d \cdot n}{D \cdot \alpha}$

$\mu 0 := 4 \cdot \pi \cdot 10^{-7}$ *

$\rho 20 := 0.0179 \cdot 10^{-6}$ *
copper @ 20 °C
partly stretched

The built, calculated and measured coils

$n := 3$	$D := 0.0251$	$ls := 0.00795$	$d := 0.002$	$f := 25.16 \cdot 10^{6}$	$la := 0.054$	$Tamb := 20.1$
$n := 4$	$D := 0.0251$	$ls := 0.0113$	$d := 0.002$	$f := 25.16 \cdot 10^{6}$	$la := 0.065$	$Tamb := 20.1$
$n := 5$	$D := 0.0323$	$ls := 0.01525$	$d := 0.002$	$f := 25.16 \cdot 10^{6}$	$la := 0.045$	$Tamb := 20.1$
$n := 7$	$D := 0.0435$	$ls := 0.02319$	$d := 0.002$	$f := 7.958 \cdot 10^{6}$	$la := 0.052$	$Tamb := 20.1$
$n := 9$	$D := 0.024$	$ls := 0.0222$	$d := 0.00132$	$f := 7.958 \cdot 10^{6}$	$la := 0.072$	$Tamb := 20$
$n := 14$	$D := 0.0837$	$ls := 0.0499$	$d := 0.002$	$f := 2.516 \cdot 10^{6}$	$la := 0.132$	$Tamb := 20.3$

the lowest sample used for calculation

Adjusting ρ for measuring temperature $\rho := \rho 20 \cdot \left[1 + (Tamb - 20) \cdot 3.9 \cdot 10^{-3}\right] = 17.921 \times 10^{-9}$

skin effect: $\delta := \sqrt{\frac{\rho}{\pi \cdot \mu 0}} \cdot \frac{1}{\sqrt{f}} = 42.476 \times 10^{-6}$

Formfactor of real coil $\alpha := \frac{ls}{D} = 0.596$ Wicklungdichte der reellen Spule $\eta := \frac{n \cdot d}{ls} = 0.561$

Calculation of the inductance

Proxieffect: G3YNH says D gets smaller for the current-tsheet -model.
The length must get longer as well. The factor sqroot 2 is guesswork.

$$Deff := D - d \cdot \left(\frac{n-2}{n} \cdot \frac{2 \cdot \eta}{1+\eta} + \frac{1-\eta}{1+\eta} \cdot \frac{d}{D}\right) = 82.45 \times 10^{-3}$$

$$lseff := ls + \frac{d}{\sqrt{2}} \cdot \left(\frac{n-2}{n} \cdot \frac{2 \cdot \eta}{\eta+1} + \frac{1-\eta}{1+\eta} \cdot \frac{d}{D}\right) = 50.781 \times 10^{-3}$$

Therefore new α and η. αeff is used for currentsheet formula $\alpha eff := \frac{lseff}{Deff} = 0.616$ $\eta eff := \frac{d \cdot n}{Deff \cdot \alpha eff} = 0.551$

with Wheeler 2 /Rosenbaum we get:

$$Lcs := \frac{\mu 0 \cdot \pi}{4} \cdot \frac{Deff}{\alpha eff} \cdot n^{2} \cdot \left[0.63667 \cdot \alpha eff \cdot \left[\ln\left(1 + \frac{1.5708}{\alpha eff}\right) + \frac{1}{\left(2.303 + 3.213 \cdot \alpha eff + 1.786 \cdot \alpha eff^{2}\right)}\right]\right] = 14.915 \times 10^{-6}$$

Correction for round wire with spacing

$$L1 := Lcs - \mu0 \cdot \frac{D}{2} \cdot n \cdot \left[\ln(2.241 \cdot \eta eff) + 0.3289 \cdot 0.1265^{\left(\frac{1}{n}\right)} - 0.0593 \cdot \left(d \cdot \sqrt{f}\right)^{-0.863} \right] = 14.57 \times 10^{-6}$$

Correction for loop when measuring the coil

An externel rectangular loop is created when the coil is connected to a measuring device. Its length is given by the length of the coil and the width is given by the length of the connecting wire plus half of the diameter of the coil.

Inductance of the wires la $$La := 4 \cdot 10^{-7} \cdot la \cdot \ln\left(\frac{2 \cdot ls}{d}\right) = 206.449 \times 10^{-9}$$ $$L := L1 + La = 14.77 \times 10^{-6}$$

Calculation of the Q-factor using Medhurst and G3YNH

$$Rdc := \frac{4 \cdot D \cdot n \cdot \rho}{d^2} = 21.000 \times 10^{-3} \quad Rskin := \frac{D \cdot n \cdot \rho}{\delta \cdot (d - \delta)} = 252.559 \times 10^{-3}$$

Interpolation of the table given by Medhurst

$$alfa := \begin{pmatrix} 0 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \\ 1 \\ 2 \\ 4 \end{pmatrix} \quad eta := \begin{pmatrix} 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \\ 0.8 \\ 0.9 \\ 1 \end{pmatrix} \quad M\Phi := \begin{pmatrix} 1.16 & 1.2 & 1.44 & 1.74 & 2.12 & 2.74 & 3.73 & 5.31 \\ 1.19 & 1.29 & 1.48 & 1.77 & 2.2 & 2.83 & 3.84 & 5.45 \\ 1.21 & 1.33 & 1.54 & 1.83 & 2.28 & 2.97 & 3.99 & 5.65 \\ 1.22 & 1.38 & 1.6 & 1.89 & 2.38 & 3.1 & 4.11 & 5.8 \\ 1.23 & 1.42 & 1.64 & 1.92 & 2.44 & 3.2 & 4.17 & 5.8 \\ 1.24 & 1.45 & 1.67 & 1.94 & 2.47 & 3.17 & 4.1 & 5.55 \\ 1.28 & 1.5 & 1.74 & 1.98 & 2.32 & 2.74 & 3.36 & 4.1 \\ 1.32 & 1.54 & 1.78 & 2.01 & 2.27 & 2.6 & 3.05 & 3.54 \end{pmatrix}$$

$$Mxy := erweitern(sort(alfa), sort(eta)) \quad coef := kspline(Mxy, M\Phi) \quad fit(\alpha, \eta) := interp\left[coef, Mxy, M\Phi, \begin{pmatrix} \alpha \\ \eta \end{pmatrix}\right]$$

$$\Phi m := fit(\alpha, \eta) = 1.760$$

considering the reduced effect of the end windings according to G3YNH.

$$Rac := Rdc \cdot \left[1 + \left(\frac{Rskin}{Rdc} - 1\right) \cdot \left[\Phi m \cdot \left(\frac{n-1}{n}\right) + \frac{1}{n}\right]\right] = 415.924 \times 10^{-3}$$

Resistance or the connecting wires of the coil. device. With skineffect only. (no proxi)

$$Rla := \frac{\rho \cdot 2 \cdot la}{\delta \cdot (d - \delta) \cdot \pi} = 18.112 \times 10^{-3}$$

Total serial loss resistance $$Rs := Rac + Rla \qquad Rs = 434.04 \times 10^{-3}$$

and now the Q-factor: $$Q := \frac{2 \cdot \pi \cdot f \cdot L}{Rs} \qquad Q = 538$$

Parallel capcitance

Approximation of Medhurst's table $$C01 := 10^{-10} \cdot D \cdot \left(0.026 + 0.14 \cdot \alpha + \frac{0.29}{\sqrt{\alpha}}\right) = 4.06 \times 10^{-12}$$

Capacitance of the hot connecting wire $$C02 := la \cdot \left(63.38 \cdot 10^{-12} \cdot d^{0.29}\right) = 1.38 \times 10^{-12}$$

$$C0 := C01 + C02 \qquad C0 = 5.44 \times 10^{-12}$$

first resonance frequency $$f0 := \frac{1}{2 \cdot \pi \cdot \sqrt{L \cdot C0}} \qquad f0 = 17.754 \times 10^{6}$$

Comparison Calculation / Measurement with HP 4342A

n	L calc. [µH]	L meas.	L calc/ meas	Q calc	Q meas	Q calc/meas
3	0.292	0.288	+ 1.4 %	368	372	- 1.1%
4	0.431	0.423	+ 2.0 %	422	429	- 1.7%
5	0.798	0.810	- 1.5 %	576	572	+ 0.7 %
7	1.98	2.039	- 3.0 %	470	463	+ 1.5 %
9	1.35	1.373	- 1.9 %	294	297	- 1.0 %
14	14.77	14.86	- 0.6%	538	529	+ 1.7 %

Measuring uncertancies are considerable. Even the mechanical data are difficult to obtain using a slide caliper. The temperature has a considerable effect also. One deg error gives 0.4% error in Q. The HP 4342A provides only limited accuracy although it is better then the specs promise.

Anyway, the calculation agrees quite well with the measurements. That means the calculation is most probably correct. It can be used to find the Q-optimum for a given volume of a coil.