Supplementary information for Grover, "Inductance Calculations" (1946).
GMDs of equal parallel rectangles.
Re-calculation of Grover's tables 1, 2, and 3, using Mathematica.
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(* This notebook shows the computations for the GMD of two rectangles, giving the results
 Grover presents in tables 1, 2 and 3, starting on page 19.  One of the few substantial errors
 in Grover is found at the very bottom of column two in table 2.  Grover has the value 1.1075,
but the correct value is 1.0748; this is a 3% error.  Grover says at the bottom of page 18,
"The general formula for log_eR is known, but involves many terms and is ill suited for computations."
 That's an understatement!  The formula is given just below.
  Usually the errors in Grover's tables are just a single count in the last place of a number;
obviously a simple rounding error.  But table 2 has a number of errors that are more than one
 count off.                                                                              *)
```

$$
\begin{aligned}
&f[b\_, c\_, p\_] := \\
&\text{If}\Big[p == c, \\
&\quad \frac{1}{24\, b^2\, c^2}\Big(2\, b\, c\,\big(b^2\,\pi - 3\, c^2\,\pi - 25\, b\, c\big) + 8\,\big(2\, b^3\, c + 15\, b\, c^3\big)\,\text{ArcTan}\Big[\frac{b}{2\, c}\Big] - 20\, b\, c\,\big(b^2 + 3\, c^2\big)\,\text{ArcTan}\Big[\frac{b}{c}\Big] - \\
&\qquad 2\,\Big(\big(18\, b^3\, c + 22\, b\, c^3\big)\,\text{ArcTan}\Big[\frac{c}{b}\Big] - 4\,\big(4\, b^3\, c + 7\, b\, c^3\big)\,\text{ArcTan}\Big[\frac{2\, c}{b}\Big] + b^4\,\text{Log}[b] - 2\, c^4\,(\text{Log}[256] - 7\,\text{Log}[c]) - \big(b^4 - 6\, b^2\, c^2 + c^4\big)\,\text{Log}\big[b^2 + c^2\big]\Big) - \big(b^4 - 24\, b^2\, c^2 + 16\, c^4\big)\,\text{Log}\big[b^2 + 4\, c^2\big]\Big), \\
&\quad \frac{1}{48\, b^2\, c^2}\Big(-100\, b^2\, c^2 + 8\, b\,(c - p)\,\big(2\, b^2 + 6\, c^2 - 12\, c\, p + 9\, p^2\big)\,\text{ArcTan}\Big[\frac{b}{c - p}\Big] + 8\, b\,(c - p)\,\big(4\, b^2 + 4\, c^2 - 8\, c\, p + 7\, p^2\big)\,\text{ArcTan}\Big[\frac{c - p}{b}\Big] + 24\, b\, c^3\,\text{ArcTan}\Big[\frac{b}{p}\Big] - 32\, b^3\, p\,\text{ArcTan}\Big[\frac{b}{p}\Big] - 120\, b\, c^2\, p\,\text{ArcTan}\Big[\frac{b}{p}\Big] + \\
&\qquad 96\, b\, c\, p^2\,\text{ArcTan}\Big[\frac{b}{p}\Big] - 144\, b\, p^3\,\text{ArcTan}\Big[\frac{b}{p}\Big] + 24\, b\, c^3\,\text{ArcTan}\Big[\frac{p}{b}\Big] - 64\, b^3\, p\,\text{ArcTan}\Big[\frac{p}{b}\Big] - 120\, b\, c^2\, p\,\text{ArcTan}\Big[\frac{p}{b}\Big] + 96\, b\, c\, p^2\,\text{ArcTan}\Big[\frac{p}{b}\Big] - 112\, b\, p^3\,\text{ArcTan}\Big[\frac{p}{b}\Big] + 16\, b^3\, c\,\text{ArcTan}\Big[\frac{b}{c + p}\Big] + \\
&\qquad 24\, b\, c^3\,\text{ArcTan}\Big[\frac{b}{c + p}\Big] + 16\, b^3\, p\,\text{ArcTan}\Big[\frac{b}{c + p}\Big] + 72\, b\, c^2\, p\,\text{ArcTan}\Big[\frac{b}{c + p}\Big] + 72\, b\, c\, p^2\,\text{ArcTan}\Big[\frac{b}{c + p}\Big] + 72\, b\, p^3\,\text{ArcTan}\Big[\frac{b}{c + p}\Big] + 32\, b^3\, c\,\text{ArcTan}\Big[\frac{c + p}{b}\Big] + 8\, b\, c^3\,\text{ArcTan}\Big[\frac{c + p}{b}\Big] + 32\, b^3\, p\,\text{ArcTan}\Big[\frac{c + p}{b}\Big] + \\
&\qquad 24\, b\, c^2\, p\,\text{ArcTan}\Big[\frac{c + p}{b}\Big] + 24\, b\, c\, p^2\,\text{ArcTan}\Big[\frac{c + p}{b}\Big] + 56\, b\, p^3\,\text{ArcTan}\Big[\frac{c + p}{b}\Big] - 2\, b^4\,\text{Log}\big[b^2 + (c - p)^2\big] + 12\, b^2\, c^2\,\text{Log}\big[b^2 + (c - p)^2\big] - 2\, c^4\,\text{Log}\big[b^2 + (c - p)^2\big] - 24\, b^2\, c\, p\,\text{Log}\big[b^2 + (c - p)^2\big] + \\
&\qquad 8\, c^3\, p\,\text{Log}\big[b^2 + (c - p)^2\big] + 12\, b^2\, p^2\,\text{Log}\big[b^2 + (c - p)^2\big] - 12\, c^2\, p^2\,\text{Log}\big[b^2 + (c - p)^2\big] + 8\, c\, p^3\,\text{Log}\big[b^2 + (c - p)^2\big] - 2\, p^4\,\text{Log}\big[b^2 + (c - p)^2\big] - 7\, c^4\,\text{Log}\Big[\frac{1}{(c - p)^2}\Big] + 24\, c^3\, p\,\text{Log}\Big[\frac{1}{(c - p)^2}\Big] - \\
&\qquad 36\, c^2\, p^2\,\text{Log}\Big[\frac{1}{(c - p)^2}\Big] + 28\, c\, p^3\,\text{Log}\Big[\frac{1}{(c - p)^2}\Big] - 9\, p^4\,\text{Log}\Big[\frac{1}{(c - p)^2}\Big] - 8\, p^4\,\text{Log}[p] - 10\, c^4\,\text{Log}[-c + p] + 32\, c^3\, p\,\text{Log}[-c + p] - 48\, c^2\, p^2\,\text{Log}[-c + p] + 40\, c\, p^3\,\text{Log}[-c + p] - 14\, p^4\,\text{Log}[-c + p] + \\
&\qquad 4\, c^4\,\text{Log}[c + p] + 16\, c^3\, p\,\text{Log}[c + p] + 24\, c^2\, p^2\,\text{Log}[c + p] + 16\, c\, p^3\,\text{Log}[c + p] + 4\, p^4\,\text{Log}[c + p] + 4\, b^4\,\text{Log}\big[b^2 + p^2\big] - 24\, b^2\, p^2\,\text{Log}\big[b^2 + p^2\big] + 4\, p^4\,\text{Log}\big[b^2 + p^2\big] - 2\, b^4\,\text{Log}\big[b^2 + (c + p)^2\big] + \\
&\qquad 12\, b^2\, c^2\,\text{Log}\big[b^2 + (c + p)^2\big] - 2\, c^4\,\text{Log}\big[b^2 + (c + p)^2\big] + 24\, b^2\, c\, p\,\text{Log}\big[b^2 + (c + p)^2\big] - 8\, c^3\, p\,\text{Log}\big[b^2 + (c + p)^2\big] + 12\, b^2\, p^2\,\text{Log}\big[b^2 + (c + p)^2\big] - 12\, c^2\, p^2\,\text{Log}\big[b^2 + (c + p)^2\big] - 8\, c\, p^3\,\text{Log}\big[b^2 + (c + p)^2\big] - \\
&\qquad 2\, p^4\,\text{Log}\big[b^2 + (c + p)^2\big]\Big)\Big]
\end{aligned}
$$

$$
g[b\_, c\_] := \frac{1}{2}\,\text{Log}\big[b^2 + c^2\big] - \frac{1}{12}\,\frac{b^2}{c^2}\,\text{Log}\Big[1 + \frac{c^2}{b^2}\Big] - \frac{1}{12}\,\frac{c^2}{b^2}\,\text{Log}\Big[1 + \frac{b^2}{c^2}\Big] + \frac{2}{3}\,\frac{b}{c}\,\text{ArcTan}\Big[\frac{c}{b}\Big] + \frac{2}{3}\,\frac{c}{b}\,\text{ArcTan}\Big[\frac{b}{c}\Big] - \frac{25}{12}
$$

This is table 1, page 19 of Grover:

```
c = 1;
Table[f[y / 10, c, 100 / x] - Log[100 / x], {x, 5, 100, 5}, {y, 1 / 100 000 000, 10 + 1 / 100 000 000}];
PaddedForm[MatrixForm[IntegerPart[% * 10 000 + 1/2] / 10 000.], {4, 4}]
```

-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
-0.0007	-0.0007	-0.0007	-0.0007	-0.0006	-0.0005	-0.0004	-0.0003	-0.0002	-0.0001	0.0000
-0.0018	-0.0018	-0.0017	-0.0016	-0.0015	-0.0013	-0.0011	-0.0009	-0.0006	-0.0003	0.0000
-0.0033	-0.0032	-0.0031	-0.0030	-0.0027	-0.0024	-0.0020	-0.0016	-0.0011	-0.0005	0.0000
-0.0052	-0.0051	-0.0050	-0.0047	-0.0043	-0.0038	-0.0032	-0.0026	-0.0018	-0.0009	0.0000
-0.0075	-0.0075	-0.0072	-0.0068	-0.0063	-0.0056	-0.0047	-0.0037	-0.0026	-0.0013	0.0001
-0.0104	-0.0103	-0.0099	-0.0094	-0.0086	-0.0077	-0.0065	-0.0051	-0.0035	-0.0017	0.0001
-0.0137	-0.0135	-0.0131	-0.0124	-0.0114	-0.0101	-0.0085	-0.0067	-0.0046	-0.0023	0.0002
-0.0175	-0.0173	-0.0168	-0.0158	-0.0145	-0.0129	-0.0108	-0.0085	-0.0058	-0.0028	0.0003
-0.0219	-0.0216	-0.0209	-0.0197	-0.0181	-0.0160	-0.0135	-0.0105	-0.0072	-0.0035	0.0005
-0.0268	-0.0265	-0.0256	-0.0241	-0.0221	-0.0195	-0.0164	-0.0127	-0.0086	-0.0041	0.0007
-0.0324	-0.0320	-0.0309	-0.0291	-0.0266	-0.0234	-0.0196	-0.0152	-0.0102	-0.0048	0.0010
-0.0386	-0.0382	-0.0368	-0.0346	-0.0315	-0.0276	-0.0230	-0.0178	-0.0119	-0.0054	0.0014
-0.0457	-0.0451	-0.0434	-0.0407	-0.0370	-0.0323	-0.0268	-0.0206	-0.0136	-0.0061	0.0019
-0.0535	-0.0528	-0.0508	-0.0475	-0.0430	-0.0374	-0.0309	-0.0236	-0.0155	-0.0068	0.0024
-0.0624	-0.0615	-0.0590	-0.0550	-0.0496	-0.0430	-0.0353	-0.0268	-0.0174	-0.0074	0.0030
-0.0724	-0.0713	-0.0682	-0.0633	-0.0568	-0.0490	-0.0400	-0.0301	-0.0194	-0.0080	0.0038
-0.0838	-0.0824	-0.0785	-0.0725	-0.0647	-0.0555	-0.0450	-0.0336	-0.0214	-0.0086	0.0046
-0.0972	-0.0953	-0.0902	-0.0827	-0.0733	-0.0624	-0.0503	-0.0373	-0.0235	-0.0092	0.0055
-0.1136	-0.1105	-0.1036	-0.0941	-0.0827	-0.0699	-0.0560	-0.0412	-0.0257	-0.0097	0.0065

This is the top half of table 2, page 20 of Gover:

```
b = 1;
Table[f[b, y / 10, 10 / x] - Log[10 / x], {x, 1, 10}, {y, 1 / 100 000 000, 10 + 1 / 100 000 000}];
PaddedForm[MatrixForm[IntegerPart[% * 10 000 + 1/2] / 10 000.], {4, 4}]
```

0.0008	0.0008	0.0008	0.0008	0.0007	0.0006	0.0005	0.0004	0.0003	0.0002	0.0000
0.0033	0.0033	0.0032	0.0030	0.0028	0.0025	0.0021	0.0017	0.0012	0.0006	0.0000
0.0074	0.0073	0.0071	0.0067	0.0062	0.0056	0.0048	0.0038	0.0027	0.0015	0.0001
0.0129	0.0128	0.0124	0.0118	0.0109	0.0098	0.0084	0.0068	0.0049	0.0027	0.0002
0.0199	0.0197	0.0191	0.0182	0.0169	0.0152	0.0131	0.0106	0.0077	0.0043	0.0005
0.0281	0.0278	0.0271	0.0258	0.0239	0.0216	0.0187	0.0152	0.0111	0.0064	0.0010
0.0374	0.0371	0.0361	0.0344	0.0320	0.0290	0.0252	0.0206	0.0152	0.0090	0.0019
0.0477	0.0473	0.0461	0.0440	0.0410	0.0372	0.0325	0.0267	0.0200	0.0121	0.0030
0.0589	0.0584	0.0569	0.0544	0.0509	0.0463	0.0405	0.0336	0.0254	0.0158	0.0046
0.0708	0.0702	0.0685	0.0656	0.0614	0.0560	0.0492	0.0411	0.0313	0.0199	0.0065

This is the bottom half of table 2, page 20 of Gover:

```
b = 1;
PaddedForm[MatrixForm[Table[IntegerPart[(f[b, y / 10, x / 10] - Log[x / 10]) * 10 000 + 1/2] / 10 000., {x, 9, 1, -1}, {y, 1 / 100 000 000, x + 1 / 100 000 000}]], {5, 4}]
```

{ 0.0848,	0.0841,	0.0821,	0.0787,	0.0738,	0.0675,	0.0596,	0.0500,	0.0386,	0.0250}
{ 0.1031,	0.1023,	0.0999,	0.0959,	0.0903,	0.0828,	0.0735,	0.0621,	0.0485}	
{ 0.1277,	0.1268,	0.1240,	0.1192,	0.1125,	0.1037,	0.0925,	0.0788}		
{ 0.1618,	0.1607,	0.1573,	0.1517,	0.1436,	0.1329,	0.1194}			
{ 0.2107,	0.2093,	0.2053,	0.1984,	0.1885,	0.1754}				
{ 0.2843,	0.2826,	0.2776,	0.2691,	0.2567}					
{ 0.4024,	0.4003,	0.3940,	0.3831}						
{ 0.6132,	0.6105,	0.6021}							
{ 1.0787,	1.0748}								

This is the rest of table 2, page 21 of Gover:

```
b = 1;
PaddedForm[MatrixForm[Table[IntegerPart[(f[b, y, x] - Log[b]) * 10 000 + 1/2] / 10 000., {x, 5/100, 5/10, 5/100}, {y, 1/100 000 000, x + 1/100 000 000, 5/100}]], {5, 4}]
```

```
{-1.3541, -1.3553}
{-1.2238, -1.2247, -1.2277}
{-1.1051, -1.1059, -1.1083, -1.1124}
{-0.9961, -0.9968, -0.9988, -1.0023, -1.0072}
{-0.8952, -0.8958, -0.8976, -0.9006, -0.9048, -0.9104}
{-0.8014, -0.8020, -0.8035, -0.8062, -0.8099, -0.8147, -0.8208}
{-0.7139, -0.7144, -0.7158, -0.7181, -0.7214, -0.7257, -0.7310, -0.7374}
{-0.6319, -0.6324, -0.6336, -0.6357, -0.6386, -0.6424, -0.6471, -0.6528, -0.6595}
{-0.5549, -0.5553, -0.5564, -0.5583, -0.5609, -0.5643, -0.5685, -0.5736, -0.5795, -0.5864}
{-0.4824, -0.4827, -0.4837, -0.4854, -0.4878, -0.4908, -0.4946, -0.4992, -0.5045, -0.5106, -0.5176}
```

This is table 3, page 22 of Grover:

```
c = 1;
T = Join[{{N[Exp[g[1/40, c]] / (1/40 + c), 5], N[Log[Exp[g[1/40, c]] / (1/40 + c)] + 3/2, 2]}}, Table[{N[Exp[g[b, c]] / (b + c), 5], N[Log[Exp[g[b, c]] / (b + c)] + 3/2, 3]}, {b, 5/100, 9/10, 5/100}],
  Table[{N[Exp[g[b, c]] / (b + c), 6], N[Log[Exp[g[b, c]] / (b + c)] + 3/2, 3]}, {b, 95/100, 1, 5/100}]];
MatrixForm[T]
```

```
0.22333  0.00089
0.22345  0.00145
0.22360  0.00210
0.22366  0.00239
0.22369  0.00249
0.22369  0.00249
0.22368  0.00244
0.22366  0.00236
0.22364  0.00228
0.22362  0.00219
0.22360  0.00211
0.22358  0.00204
0.22357  0.00197
0.22356  0.00192
0.22355  0.00187
0.22354  0.00184
0.22353  0.00181
0.22353  0.00179
0.22353  0.00178
0.223525 0.00177
0.223525 0.00177
```

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