Grover's 'Inductance Calculations'
Supplementary information and errata

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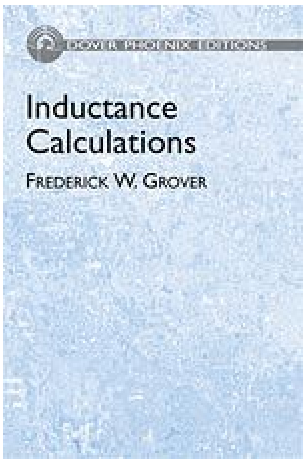

Inductance Calculations: Working Formulas and Tables
F W Grover, 1946 and 1973.
Dover Phoenix Edition 2004. ISBN: 0 486 49577 9.
2009 reprint ISBN13: 9780486474403
Dr Frederick W Grover's monograph for engineers and scientists engaged in the accurate calculation of self and mutual inductance. The book is based on the work carried out by E B Rosa and F W Grover during their distinguished careers at the American National Bureau of Standards during the first half of the 20th Century. Much of the information is given in tabular form; as befits the calculation methods used at the time, but source material is fully referenced, and most of the generating functions are presented and explained.

Contents:

Errata and supplementary data

Errors and their corrections are shown below in the form of scanned pages marked in pencil. Most of these were discovered through the careful work of Rodger Rosenbaum:
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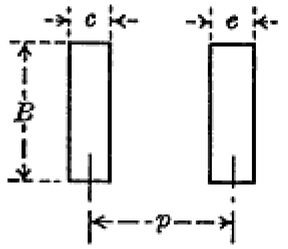
TABLE 2. GEOMETRIC MEAN DISTANCES OF EQUAL PARALLEL RECTANGLES

Values of $\log_e k$ in equation $\log_e R = \log_e p + \log_e k$. (b) Longer sides of rectangles perpendicular to line joining their centers.



$$\beta = \frac{B}{p}, \quad \Delta = \frac{C}{B}.$$

β	$\Delta = 0$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	0	0	0	0	0	0	0	0	0	0	0
0.1	0.0008	0.0008	0.0008	0.0008	0.0007	0.0006	0.0005	0.0004	0.0003	0.0002	0.0000
.2	.0033	.0033	.0032	.0030	.0028	.0025	.0021	.0017	.0012	.0007	.0000
.3	.0074	.0073	.0071	.0067	.0062	.0056	.0048	.0038	.0027	.0015	.0001
.4	.0129	.0128	.0124	.0118	.0109	.0098	.0084	.0068	.0050	.0027	.0003
0.5	0.0199	0.0197	0.0191	0.0182	0.0169	0.0152	0.0131	0.0106	0.0077	0.0043	0.0005
.6	.0281	.0278	.0271	.0258	.0240	.0216	.0185	.0152	.0111	.0064	.0011
.7	.0374	.0371	.0361	.0344	.0320	.0290	.0251	.0206	.0155	.0090	.0019
.8	.0477	.0473	.0461	.0440	.0411	.0373	.0321	.0268	.0200	.0129	.0031
0.9	.0589	.0584	.0569	.0544	.0506	.0464	.0404	.0338	.0254	.0158	.0046
1.0	0.0708	0.0702	0.0685	0.0655	0.0614	0.0560	0.0492	0.0406	0.0313	0.0199	0.0065
0.9	.0847	.0841	.0821	.0787	.0738	.0675	.0596	.0501	.0382	0.0250	
.8	.1031	.1023	.0999	.0959	.0903	.0829	.0745	.0622	0.0485		
.7	.1277	.1268	.1240	.1192	.1125	.1037	.0925	0.0788			
.6	.1618	.1607	.1573	.1507	.1436	.1329	0.1194				
0.5	0.2107	0.2094	0.2053	0.1984	0.1886	0.1754					
.4	.2843	.2826	.2776	.2691	0.2567						
.3	.4024	.4003	.3942	0.3831							
.2	0.6132	0.6105	0.6021								
0.1	1.0787	1.1075									
$1/\beta$											

1.0748

FIG. 9

Included in Table 1 is the case shown in Fig. 9 which may be calculated directly by the simple relation [19]

$$\log_e R = \log_e na - \left(\frac{1}{12n^2} + \frac{1}{60n^4} + \frac{1}{168n^6} + \frac{1}{360n^8} + \frac{1}{660n^{10}} + \cdots\right). \quad \text{(e)}$$

This is very convergent for all values of n except $n = 1$. For that case, Table 1 gives $\log_e R = \log_e a - 0.1137$.

A complete recalculation of tables 1, 2, and 3 has been given by Rodger Rosenbaum[1]. Most of the errors are small except for the one shown above, which should be marked on any working copy.

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34 CALCULATION OF MUTUAL INDUCTANCE AND SELF-INDUCTANCE

In the rare cases where the distance between the filaments is large compared with their lengths, the following series development is simple and accurate:

$$M = 0.002l\left(\frac{1}{2}\frac{l}{d}\right)\left[1 - \frac{1}{12}\frac{l^2}{d^2} + \frac{1}{40}\frac{l^4}{d^4} - \cdots\right]. \qquad (4)$$

Formulas (2), (3), and (4) cover all possible cases.

Example 1: The mutual inductance of two filaments each 10 feet long, spaced 6 inches apart, is required.

The length is $l = 10 \times 12 \times 2.54 = 304.8$ cm., $\frac{d}{l} = \frac{6}{120} = 0.05$, $\frac{2l}{d} = 40$. From Auxiliary Table 1, page 236, $\log_e 40 = 3.6889$.

Therefore, from (3), $M = 0.002(304.8)[3.6889 - 1 + 0.05 - 0.00006] = 1.6690\ \mu$h.

If the same filaments were placed 3 feet apart, $\frac{d}{l} = 0.3$, and from Table 5, $Q = 1.1749$, so that $M = 0.002(304.8)(1.1749) = \cancel{0.6999}$ 0·7162 μh. This value agrees also with that found for this case by formula (3).

In the extreme case that the filaments are separated by a distance of 40 feet, $\frac{l}{d} = \frac{1}{4}$, and Table 5 leads to the value $Q = 0.1244$.

Therefore,

$$M = 0.002(304.8)(0.1244) = 0.07583\ \mu\text{h}.$$

The series formula (4) gives

$$M = 0.6096(\tfrac{1}{8})[1 - (\tfrac{1}{12})(\tfrac{1}{16}) + (\tfrac{1}{40})(\tfrac{1}{256})]$$

$$= 0.0762[0.99489] = 0.07581\ \mu\text{h}.$$

The above contains a simple arithmetic error: 0.002 x 304.8 x 3.6889 = 0.7162

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A discussion of the accuracy of Grover's tabular calculation method is given by Engel and Rohe[2]. Claims made in that paper relating to inaccuracies in Grover's method may however be flawed due to the assumption that the FEEFC calculations carried out were equivalent to actual measurements. Rodger Rosenbaum investigated this matter in April 2009, and apart from changes in the last digit of some tabulated values, no errors were found in Grover's work.

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1 **GMDs of equal parallel rectangles. Re-calculation of Grover's table 1, 2, and 3**. Rodger Rosenbaum. 2009. Available from g3ynh.info.

2 **A Comparison of Single-Layer Coaxial Coil Mutual Inductance Calculations Using Finite-Element and Tabulated Methods**, Thomas G Engel and Stacy N Rohe. IEEE Transactions on Magnetics, Vol 42, No. 9, Sept. 2006. p2159-2163.

Pages 143 - 147

SINGLE-LAYER COILS ON CYLINDRICAL WINDING FORMS 143

rent sheet of infinite length and applies a correction to take account of the effect of the ends.

Nagaoka's formula is

$$L = 0.004\pi^2 a^2 b n^2 K$$
$$= 0.002\pi^2 a \left(\frac{2a}{b}\right) N^2 K, \qquad (118)$$

in which n is the winding density in turns per centimeter of axial length and K is the factor that takes account of the effect of the ends. Nagaoka gave[74] a table of values of K as a function of the shape ratio $\frac{2a}{b} = \frac{\text{diameter}}{\text{length}}$, which is reproduced here. For relatively short coils, b less than $2a$, it is convenient to tabulate K for such coils as a function of $\frac{b}{2a}$. This has been done in Table 36. Table 37 includes Nagaoka's values of K for long coils. From this table very accurate values of K may be interpolated, sufficing for the calculation of the inductance of standard coils intended for the most precise work.

For very short coils, interpolation in Table 36 becomes uncertain and it is better to derive K directly from the following series formula:[75]

$$K = \frac{2\beta}{\pi}\left[\left(\log_e \frac{4}{\beta} - \frac{1}{2}\right) + \frac{\beta^2}{8}\left(\log_e \frac{4}{\beta} + \cancel{\frac{1}{8}}\right) \tfrac{1}{4}\right.$$
$$\left. - \frac{\beta^4}{64}\left(\log_e \frac{4}{\beta} - \frac{2}{3}\right) + \frac{5}{1024}\beta^6\left(\log_e \frac{4}{\beta} - \frac{109}{120}\right) - \cdots\right], \qquad (119)$$

in which $\beta = \frac{b}{2a}$. For values of β as large as $\frac{1}{4}$ three terms will suffice for an accuracy better than 1 part in 1000.

Formula 119 on page 143, is Coffin's formula truncated, which is in turn an extended version of the Rayleigh-Niven formula[3]. The second term in the series should be:
. . . + $(\beta^2/8)[\ln(4/\beta)$ + **1/4**] - . . .
Consideration was given to the possibility that Grover might be correcting errors in the 1911 document, but a comparison against Lundin's formula[4] indicated that ¼ was the best choice for the term in question. Rodger Rosenbaum has since compared the formula against Lorenz's exact expression, and confirms this conclusion as shown below:

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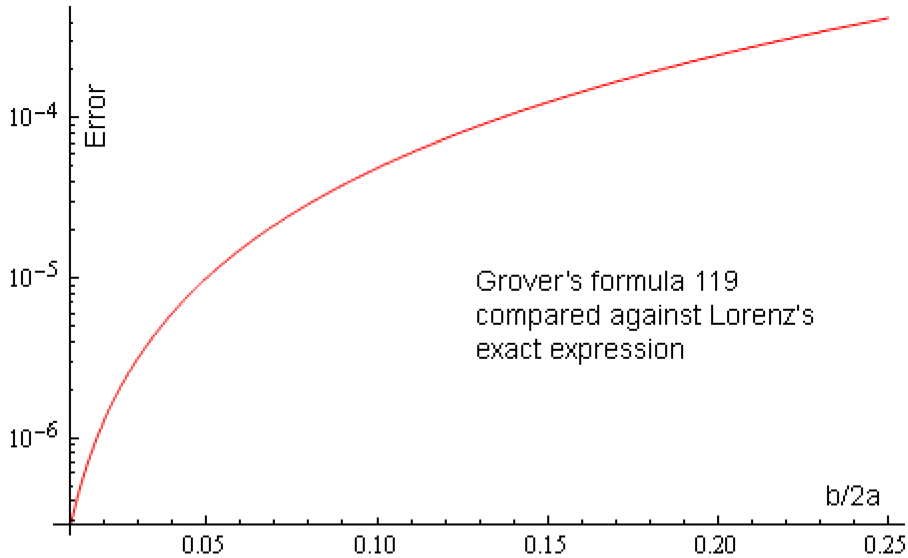
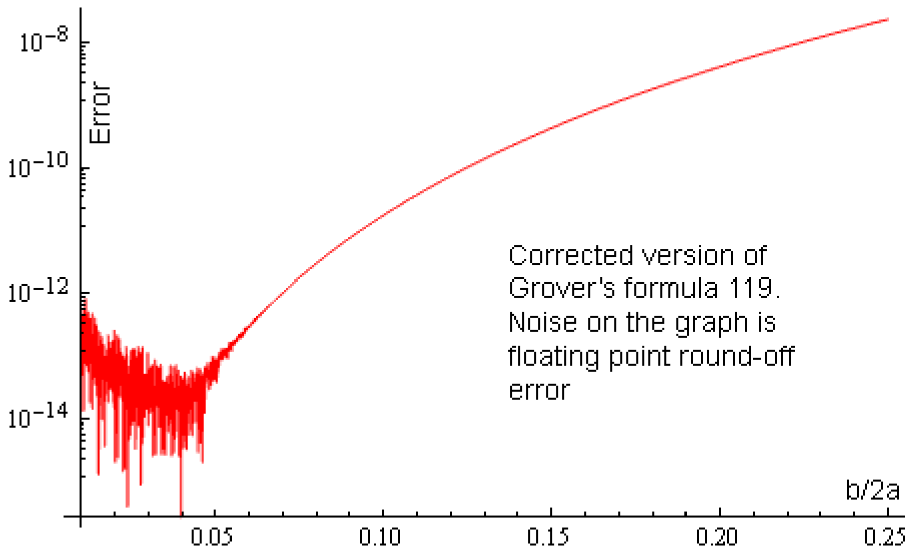
3 **Formulas and Tables for the calculation of mutual and self induction**. E B Rosa and F W Grover. 3rd edition 1916 with 1948 corrections BS Sci. 169 (available from g3ynh.info). See pages 116 - 117, formulas 69 and 71.

4 **A Handbook Formula for the Inductance of a Single-Layer Circular Coil.** R. Lundin Proc. IEEE, vol. 73, no. 9, pp. 1428-1429, Sep. 1985.





Note however, Grover's comment above: "For values of β as large as ¼ three terms will suffice for an accuracy better than 1 part in 1000". As is confirmed by Rodger's calculation, even with the error, this statement is correct. Rodger comments: "When you look at the plots, it is clear that ¼ is the right value. The error as β gets small descends right down to the round-off error of the arithmetic for a value of ¼, but not for 1/8. The fact that the error is better than 1 part in 1000, as Grover says, even with 1/8, suggests that he did some calculations with the 1/8 value, and noticed the error was 1 part in 1000, failing to notice that with ¼ the error would be much smaller."

Rodger has also calculated the exact values of Nagaoka's coefficient *K* as they should appear in tables 36 and 37 (p144-147), (see below). Some minor differences occur, and the tables can be amended using the data provided.

The exact formula for Nagaoka's coefficient is:

$$f[z] = \frac{4}{3\pi}\left(\frac{\sqrt{1+z^2}}{z^2}\left(K\left[\frac{z^2}{1+z^2}\right] - E\left[\frac{z^2}{1+z^2}\right]\right) + \sqrt{1+z^2}\,E\left[\frac{z^2}{1+z^2}\right] - z\right)$$

where K[x] and E[x] are elliptic integrals of the first and second kind respectively; and z=2a/b

0.034960	0.373818	0.530309	0.626122
0.061098	0.381986	0.535017	0.629185
0.083908	0.389944	0.539637	0.632200
0.104562	0.397703	0.544171	0.635170
0.123615	0.405269	0.548620	0.638094
0.141395	0.412651	0.552989	0.640973
0.158119	0.419856	0.557278	0.643810
0.173942	0.426890	0.561490	0.646604
0.188980	0.433761	0.565627	0.649357
0.203324	0.440475	0.569691	0.652070
0.217044	0.447036	0.573683	0.654742
0.230200	0.453451	0.577606	0.657376
0.242842	0.459724	0.581462	0.659972
0.255011	0.465860	0.585251	0.662530
0.266744	0.471865	0.588976	0.665052
0.278070	0.477741	0.592639	0.667539
0.289019	0.483495	0.596240	0.669990
0.299614	0.489128	0.599781	0.672407
0.309876	0.494646	0.603264	0.674791
0.319825	0.500051	0.606690	0.677141
0.329479	0.505348	0.610060	0.679460
0.338853	0.510539	0.613376	0.681746
0.347961	0.515628	0.616639	0.684002
0.356816	0.520617	0.619850	0.686227
0.365432	0.525510	0.623011	0.688423

Values for Grover's
Table 36
(pages 144-145)

0.995768	0.898033	0.815082	0.745190
0.991562	0.894440	0.812049	0.742637
0.987380	0.890871	0.809037	0.740100
0.983223	0.887325	0.806046	0.737581
0.979092	0.883803	0.803075	0.735079
0.974985	0.880304	0.800125	0.732594
0.970903	0.876829	0.797195	0.730126
0.966846	0.873377	0.794285	0.727674
0.962814	0.869948	0.791395	0.725239
0.958807	0.866542	0.788525	0.722820
0.954825	0.863159	0.785674	0.720418
0.950867	0.859799	0.782843	0.718032
0.946934	0.856461	0.780032	0.715662
0.943026	0.853146	0.777240	0.713308
0.939143	0.849853	0.774467	0.710969
0.935284	0.846583	0.771713	0.708646
0.931449	0.843335	0.768978	0.706339
0.927639	0.840109	0.766262	0.704047
0.923854	0.836905	0.763564	0.701770
0.920093	0.833723	0.760885	0.699508
0.916356	0.830563	0.758224	0.697262
0.912643	0.827424	0.755582	0.695030
0.908954	0.824307	0.752957	0.692813
0.905290	0.821211	0.750351	0.690611
0.901649	0.818136	0.747762	0.688423

Values for Grover's
Table 37
(pages 146-147)

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A recalculation of Rosa's mutual inductance correction factor H (Table 39, p150) to 10 decimal places, and a discussion of the GMD method, is given by Bob Weaver[5] . Grover's table has no errors greater than 1 in the last digit, but the more accurate information may be preferred. Bob's article also provides information on the coding of calculation routines. A one-line continuous formula which gives H with a maximum absolute error of better than ±0.000 000 02 is given by David Knight[6].

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The pitch of the winding is

30.5510

$$p = \frac{30,5510}{440},$$

and the ratio

$$\frac{\delta}{p} = 0.9131.$$

Interpolating for this value in Table 38, $G = 0.4659$
and from Table 39 for $N = 440$, $H = 0.3349$
so that the correction is Sum $= 0.8008$,

$$\Delta L = 0.004\pi(440)(0.8008)a$$
$$= 120.0\ \mu\text{h} = 0.0001200 \text{ henry.}$$

So, finally, the inductance of the coil is

$$L = 0.1018102 - 0.0001200 = 0.1016902 \text{ henry.}$$

A comma has been inserted instead of a decimal point: p = 30.5510

5 **Investigation of Rosa's round-wire mutual inductance correction formula**. Robert Weaver, 2008. Available from g3ynh.info.

6 **Rosa's mutual inductance correction for the round-wire solenoid**. David Knight, 2010. available from g3ynh.info.

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In this example $n_2\delta_2 = 50(0.015) = 0.75$, so that from Table 38 $G = 0.27$ and for $N = 425$ Table 39 gives $B = 0.335$. Consequently

$$\frac{\Delta L}{L} = \frac{6.28(0.27 + 0.335)}{8.17(50)(4)} = 0.0023,$$

or the correction amounts to 2.3 parts in 1000 of the whole inductance. This value may also be checked by (124), which gives

$$\frac{\Delta L}{L} = \frac{6.28(0.605)}{3.842(425)} = 0.0023.$$

The corrected inductance will therefore be 16·2 7034·8

$$7051(1 - 0.0023) = 7051 - \cancel{1.6} = \cancel{7049}\ \mu h.$$

Arithmetic error: -0.0023 x 7051 = -16.2173
7051 - 16.2 = 7034.8

Page 161

On page 161, the last line in formula 134 reads:
a=2.451b
But this has to be
2a=2.451b
The number "2" is missing. For confirmation, see line 1 of formula 134:
2a/b=2.451

(error found by Thomas Heckel, Feb. 2011)

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inches, so that, in Table 40, r is to be taken as 12 divided by 15 = 0.8. The corresponding value of F is 5.80. Thus, by formula (122a)

$$L_s = 0.00254(5.80)12(20)^2 = 70.71\ \mu h.$$

To evaluate the correction for insulation, $\frac{\delta}{p} = \frac{1}{3}$, and $N = 20$, so that from Tables 38 and 39,

$$\begin{aligned} G &= -0.5417 \\ H &= +0.2964 \\ \hline G + H &= -0.2453. \end{aligned}$$

Consequently the correction in (135) is

$$\Delta L = 0.004\pi(20)(\overset{}{\underset{6}{\cancel{12}}} \times 2.54)(-0.2453) = \overset{-0.9396}{\cancel{-1.88}}\ \mu h,$$

and the inductance of the helix is

$$L = 70.71 - (-\overset{0.9396}{\cancel{1.88}}) = \overset{71.65}{\cancel{72.6}}\ \mu h.$$

Here the error can be seen by looking back at formula 135 (p163); Grover used the diameter (12 inches) instead of the radius (6 inches). The above should read:
$\Delta L = 0.004\ \pi\ (20)(6 \times 2.54)(-0.2453) = -0.9396\ \mu H$,
and the inductance of the helix is:
$L = 70.71 - (-0.9396) = 71.65\ \mu H$.

David Knight & Rodger Rosenbaum. April 2009.
Updated March 2011, June 2012 (converted to pdf).