A self-evaluating precision reference bridge

By David Knight

An HF transmission bridge with maximum phase error of ±<0.1° and maximum impedance magnitude error of ±<0.2% over the 1.6 MHz to 30 MHz range.

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Please check the author's website to make sure you have the most recent versions of this document and its accompanying files: http://www.g3ynh.info/zdocs/bridges/ .

This article is an updated version of an HTML document that first appeared on the author's website in December 2007. Except for minor changes and corrections, the original content and conclusions are unaltered.

Abstract

The construction and evaluation of an accurate 50 Ω impedance-matching-reference bridge operating over the 1.6 MHz to 30 MHz range is described. The bridge is based on a 1:12 toroidal current transformer and has a nominal mid-band insertion loss of 0.06 dB. The maximum recommended continuous power throughput is 100 W.

Voltage sampling is by means of a capacitive potential divider. Two-point magnitude-vs.-frequency tracking is achieved by adjustment of the potential-divider ratio at the LF end of the operating range and by adjustment of an inductance compensation coil at the HF end. LF phase compensation is by means of an adjustable resistance in parallel with the lower voltage-sampling arm.

A wire passing through the transformer core is used for HF phase compensation. A quadrature current is injected into this auxiliary winding by means of an adjustable capacitor. Fine control of the phase of the correction signal is achieved by means of an adjustable resistance connected across the extra winding. This arrangement gives three-point phase-vs.-frequency tracking.

The bridge is rendered self-evaluating by choice of orthogonal adjustments for resistance and reactance balance, and by including provision for the associated adjustment parameters to be measured. The theoretical basis on which frequency-dependent deviations of the parameters from their calibration settings can be converted into performance data is given.

Both Faraday-shielded and unshielded versions of the current transformer were tried. In both cases, with the bridge maintained at constant temperature; the maximum maximum peak-to-peak phase runout was about ±0.03°; and the maximum magnitude runout was about ±0.04%. The ultimate magnitude accuracy is limited by the uncertainty in the value of the reference load resistance used during calibration. Using a 4½-digit Fluke 8060A multimeter for the resistance measurement gave a final magnitude accuracy of about ±0.13%. The results obtained constitute a two orders-of-magnitude improvement over typical Douma (CVS) transmission-bridge designs.

The effect of temperature variation on phase accuracy is investigated. This issue is related to the temperature coefficient of the permeability of the transformer core. The effect is only significant at low frequencies. A compensation scheme using a Linear PTC thermistor is proposed but not tested.

Note: The evaluation procedure, the calibrated adjustable capacitor, and the data analysis, are not needed if the intention is simply to make a working version of the bridge.

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1. Introduction

A study of the various causes-of and remedies-for phase and magnitude error in RF current-transformer bridges was made in the author's previous article on the subject of transmission bridge optimisation[1]. For a full discussion of the background theory, readers should refer to that document; but a summary of the relevant findings can be given as follows:

● For a bridge with a capacitive potential-divider for the voltage-sampling network; the major cause of amplitude error is the inductance of the lower voltage-sampling arm. Best results are obtained when:

a) The inductance of the lower voltage sampling arm is kept to a minimum.

b) The residual inductance is balanced-out by means of an adjustable inductor in series with the upper voltage-sampling arm.

● Of the various HF phase compensation schemes tried, by far the best was a quadrature current injection technique with two adjustable parameters. The method involves fitting the transformer with an auxiliary 1-turn winding. The injected current is set by means of a trimmer capacitor, and fine-tuning of the phase of the correcting signal is accomplished by means of an adjustable resistor placed across the winding. The system permits the phase error to be brought to zero at two points in the mid-to-upper frequency range.

● For a bridge corrected to give near-perfect phase and amplitude performance and having a relatively high transimpedance; there is little basis on which to choose between current-transformers with or without a Faraday shield.

This article describes the construction of a 1.6 MHz - 30 MHz 50 Ω impedance reference (match monitoring) bridge employing the corrections outlined above. During the optimisation work, the opportunity was taken to test for performance differences between a Faraday-shielded and an un-shielded version. No significant differences were found.

A problem with the production of current-transformer bridges is that of how to determine the accuracy of the finished unit. Traditionally, this has been such an awkward matter that most constructors take the performance as a matter of trust. This kind of trust is rarely warranted; and underlies such ludicrous comments as: "I get a perfect 1:1 SWR on [some frequency at the low or high end of my bridge's working range] but my transmitter refuses to give full output". Fortunately however, such privations can be avoided without recourse to exotic test-equipment. This is because a bridge can be made to reveal its own shortcomings; provided that there is a way of determining the extent to which it is out of balance at a particular frequency when connected to a reference load resistance. The trick is to design the bridge in such a way that it has a pair of adjustments capable of bringing it to balance at any frequency in its operating range, and to arrange things so that relative measurements of the values of the two adjustable components can be made without causing disruption.

A method of bridge evaluation by parameter-shift measurement was developed in the preceding article[2]. Here we refine that method. An ideal candidate for the technique is Douma's bridge [**US Pat. No. 2808566**]; which has a capacitive potential divider, and uses a resistance placed across the lower voltage-sampling arm to compensate for the finite inductance of the current-transformer

1 **Evaluation and optimisation of current transformer bridges**. [**Eval. & opt.**] D W Knight. http://g3ynh.info/zdocs/bridges/

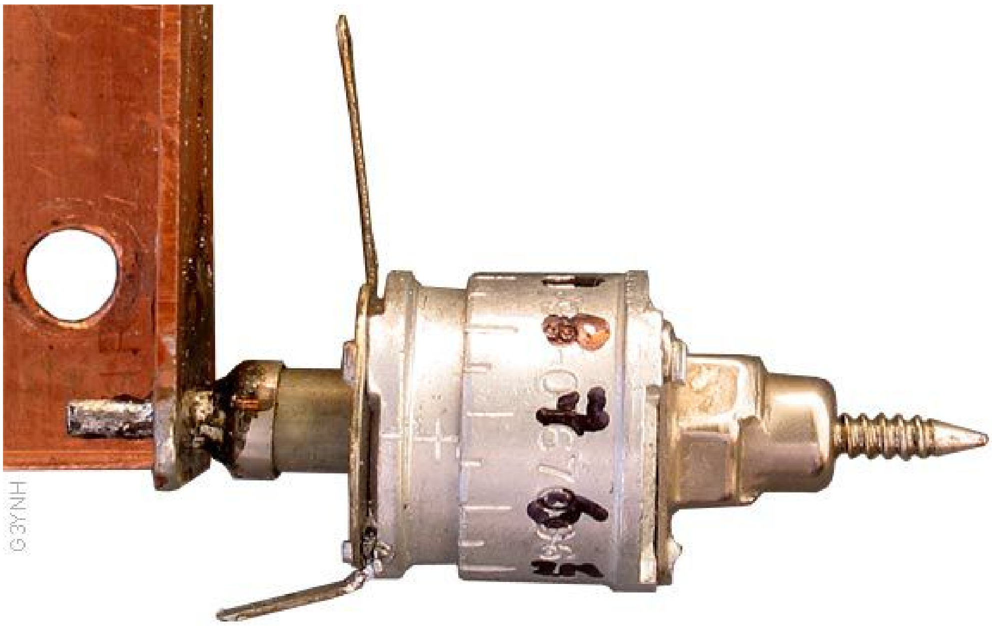
2 **Eval. & Opt.** Section 16.

secondary winding. Normally the bridge is calibrated at the low end of its operating frequency range by adjusting the resistance and one of the potential-divider capacitors; but provided that the phase error at high frequencies is not too great, these adjustments can be used to balance the bridge at any frequency. That being the case, if the bridge re-balanced at some test frequency, the deviations of the new potential divider settings from the calibration settings can be used to calculate the phase and magnitude error that would have occurred had the settings been left alone. Evaluating the bridge is then a matter of reading the resistance and capacitance values at a representative set of spot frequencies and analysing the data.

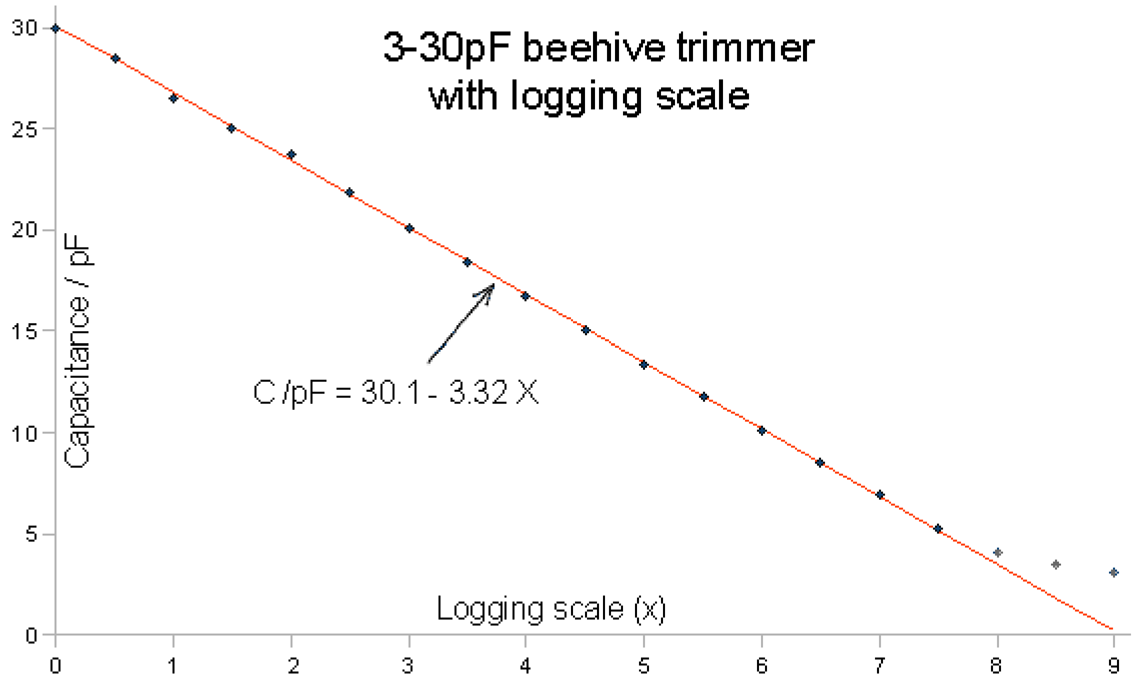
The potential divider resistance is easily determined by measuring the DC resistance looking into the detector port. This presumes that any passive detector (if used) can be unplugged. A diode detector is however, far too insensitive for accurate balance-point determination; and so, at least during the evaluation process, the detector port is connected to a radio receiver. When that is done, a DC blocking capacitor can be placed in the line to the receiver input, allowing a resistance meter to be left connected via a T-piece during the test procedure.

Measuring the potential divider capacitance is a little more difficult. For the experiments that preceded this work, a plug-in variable reference capacitor was used; but such an approach is not suitable for a working unit. The solution adopted here was to scribe logging scales onto a rotary-turret-type ("beehive") trimmer capacitor. The author used a lathe with an angle-finding attachment for that purpose, but it is also possible to improvise a suitable jig using a 360° protractor and a drill-press. Measurements of scale reading vs. capacitance are taken prior to installation and fitted to a regression function[3]. The regression formula is used to convert readings into capacitances for the data analysis.

3 See, for example, **Scientific Data Analysis**. [**Data analysis**] D W Knight. http://g3ynh.info/zdocs/math/data_analy.pdf



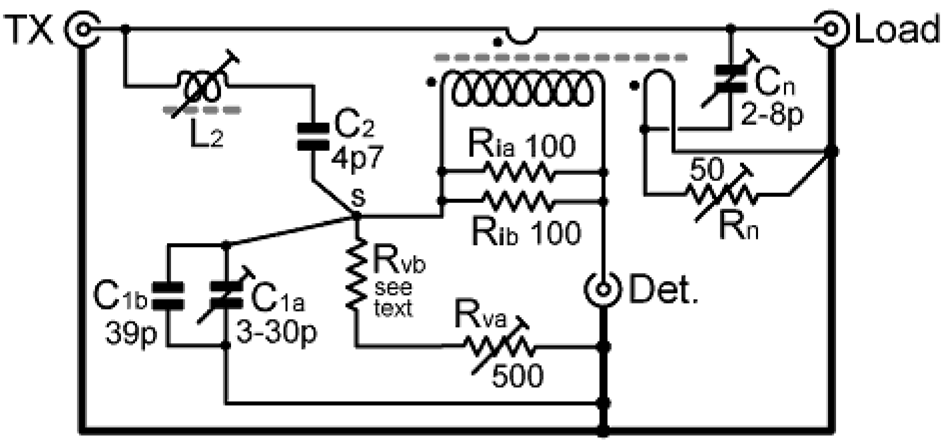
Philips 3 pF - 30 pF beehive trimmer with position logging scales added. The top part of the trimmer was mounted in a 3-jaw lathe-chuck, with a 360° protractor attached to the back-end of the main-shaft. The marks at 18° intervals were made using a scriber mounted in the tool-post. The trimmer, as shown, is 1.675 turns away from the maximum capacitance position (0 turns).



Readings of capacitance vs. logging scale were taken using a laboratory bridge and fitted to a regression line. The measurement frequency was 1.5915 MHz. Raw data are given in the open-document spreadsheet file **refbrg_D_01.ods** (sheet 3). The gradient is -3.32 pF/turn, allowing a reading resolution of about 0.016 ±0.016 pF by estimation between the markings. The data progression becomes non-linear when the two parts of the capacitor become unmeshed. Hence data beyond 7½ turns (grey) are excluded from the fit. On installation, the trimmer is padded with a fixed capacitor chosen to ensure operation within the linear region.

2. Circuit, components, and layout

The circuit and layout of the bridge is shown below. This is based on Douma's bridge [**US Pat. No. 2808566** (1953)], with the addition of a small inductor L_2 in series with the upper voltage-sampling arm, and a phase-neutralising network comprising C_n, R_n and a 1-turn auxiliary winding on the current transformer.



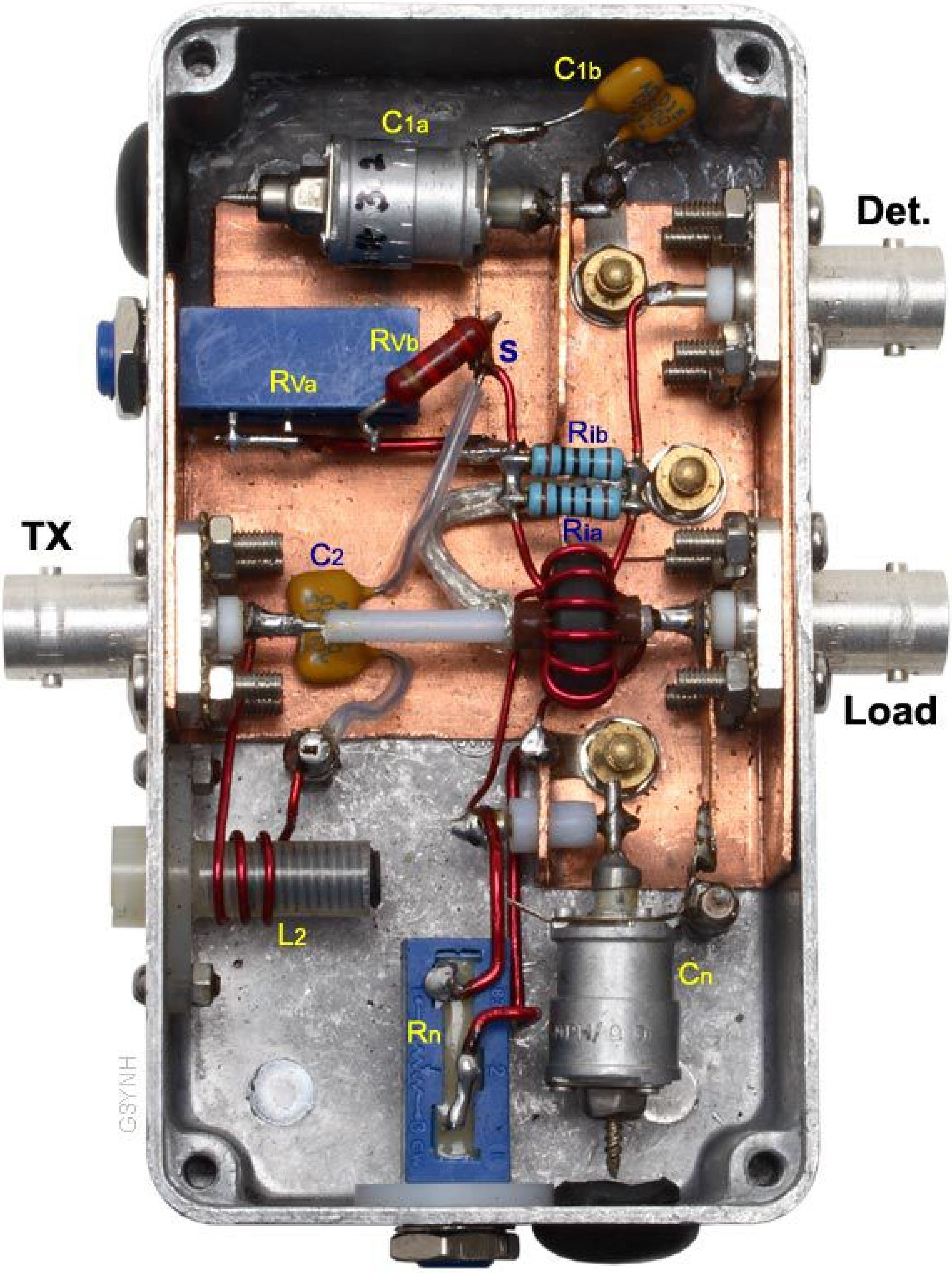
The current transformer is made by winding 12 turns of 0.9 mm diameter enamelled copper wire onto an Amidon FT50-61 ½" ferrite toroid. The A_L value of the core is 69 nH/turn² ±25% , predicting a secondary inductance of 9.9 ±2.3 µH. The measured secondary inductance of the author's transformer (L_{sec}) was 8.15 µ H at 1.5915 MHz. Due to leakage inductance, the effective or 'coupled' secondary inductance (L_i) is about 1% less than the measured value.

The bridge was tested both with and without a Faraday shield. The Faraday shielded primary was made from a stub of UR-M108 (Ag-PTFE, FEP-coated, 50 Ω) coaxial cable. The unshielded primary was a piece of 1 mm diameter silver-plated copper wire. An insulator to keep the wire centred in the transformer was cut from a short length of polyethylene honeycomb insulator as used in 75 Ω UHF-TV downlead cable (the point being to keep the dielectric constant as low as possible to minimise stray capacitance).

Note that the Faraday shield earth is taken from the generator (TX) side of the transformer core. This arrangement prevents capacitive currents from the through-line to the shield from affecting the high-frequency phase performance of the transformer[4] .

The effective secondary inductance (L_i) determines the required value of the LF compensation resistance ($R_v = R_{Va} + R_{Vb}$). For the voltage-sampling divider ratio used and $L_i = 8$ µH , it was found that 2.2 kΩ to 2.4 kΩ was a suitable choice for R_{Vb} when the transformer was fitted with an unshielded primary, and 2.4 kΩ to 2.7 kΩ when a Faraday shield was used (an unshielded bridge has additional stray capacitance across C_2 and stray coupling from the through-line to the detector port, both of which affect the balance condition). On commissioning, it is a simple matter to set the R_{Va} to the middle of its travel and connect a 5 kΩ pot. in place of R_{Vb}. Then, once the bridge has been roughly set up, the pot. can be removed, measured, and replaced by the nearest fixed resistor. The power rating of the resistor is not critical, the total dissipation in the voltage sampling network being less than 20 mW when the input to the bridge is 100 W.

4 **Eval. & Opt.** section 11.



Plan view of the precision bridge fitted with a transformer Faraday-shield

A low value for C_2 was chosen in order to keep the generator (transmitter) power-factor reasonably close to unity. For the layout used, with a Faraday shielded transformer, the effective value for C_2 is about 0.2 pF greater than the measured value of the capacitor itself, due to strays. When the bridge was tested with no Faraday shield, changes noted in the balance condition indicated additional strays, from the primary to the summing point (s), of about 0.23 pF.

The transformer core is mounted close to the load socket. This keeps the load-side line mismatch to a minimum. Mismatch on the generator (TX) side does not affect the balance condition, and it is beneficial to leave a short length of this part of the line unshielded. For a 50 Ω match, the through-line should have an average capacitance of about 94 pF/m. Removing most of the shield on the TX side subtracts about 2 pF, offsetting some of the 4.5 pF or so of voltage-sampling-network capacitance. Note that the neutralisation network also places capacitance across the generator, giving a total burden of about 7 pF. This capacitance, which gives rise to an input SWR of about 1.1 at 30 MHz, can be cancelled-out if so desired by placing additional inductance between the TX socket and the bridge circuitry (see section 7).

The transformer secondary is terminated by two 0.5 W metal-film resistors with the leads clipped to a length just sufficient for connection. The point of the arrangement is that the inductance of the termination gives rise to a positive (leading) phase error in the transformer output at high frequencies[5], whereas the neutralising network can only compensate for lagging phase errors. The total power rating of 1 W allows for a maximum secondary current of 141 mA, i.e. a maximum primary current of $0.141 \times 12 = 1.7$ A, corresponding to 144 Watts in a 50 Ω load. This gives a burnout safety-factor of 1.44 when using a standard 100 W transmitter. Carbon-film resistors, being liable to change in value when overheated, are not recommended for this position.

In construction and in the choice of components, the inductance of the lower voltage sampling arm (C_{1a}, C_{1b}, R_{Va}, R_{Vb}) must be kept to a minimum. The point is that the total inductance of the arm must resolve accurately into a frequency-independent equivalent-series-inductance (ESL) if the frequency response is to be corrected by a single inductance (L_2) in series with C_2. This requirement is also assisted by star-wiring the components to the summing point (**s**), rather than distributing the connections along a rail or circuit-track.

If L_1 is the ESL of the lower voltage-sampling arm, the compensating inductance is given by:

$$L_2 = L_1 C_1 / C_2$$

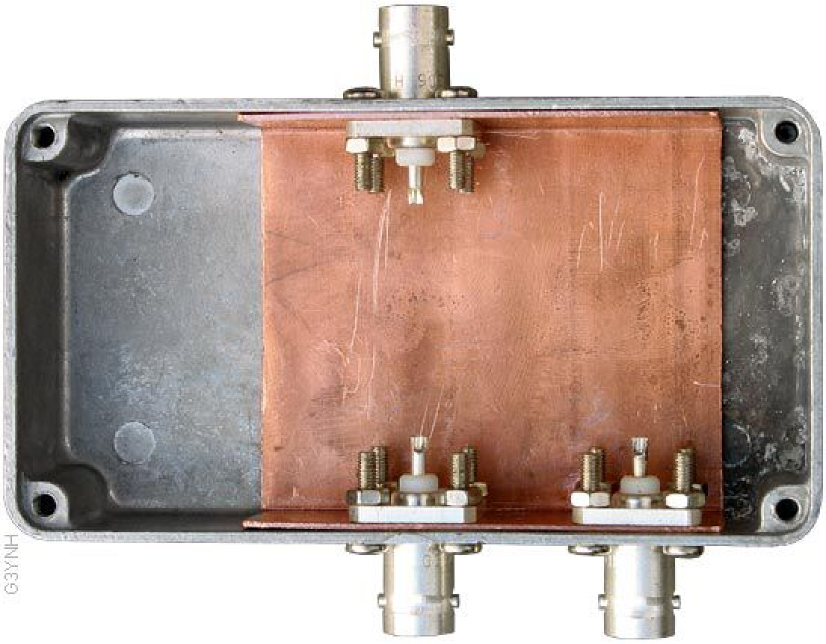
see ref.[6]. Since C_1 is about 12 times larger than C_2, there will still be a need for a compensating inductance of about 250 nH after careful minimisation of L_1. Consequently, since the inductance of C_2 is part of the total compensating inductance, there is no particular need for C_2 to have short leads. The leads simply need to be short enough to allow a positive inductance value for the coil L_2, so that there will be some latitude for inductance adjustment. For the author's layout, 3 turns of 0.9 mm diameter wire on a 7 mm former, spaced at a little more than the wire diameter, did the trick (the spacing can be adjusted to vary the inductance slightly).

The phase-neutralising trimmer C_n causes a quadrature current to flow in a 1-turn auxiliary winding. The extra winding is a piece of 0.7 mm diameter enamelled copper wire pushed through the core in the gap between the secondary lead-out wires. Since this wire is effectively at ground potential, it causes the minimum of disturbance to the secondary winding when threaded through in that position. The phasing is chosen so the the quadrature current cancels the HF phase-lag in the transformer output caused by propagation delay and load-side through-line mismatch. The auxiliary winding produces a small voltage, the phase of which can be adjusted by means of the variable shunt resistance R_n. This gives a minor adjustment of the phase of the neutralising current.

5 **Eval. & Opt.** section 11.
6 **Eval. & Opt.** section 17.

When designing for accuracy, it is important to be aware that the earth conduction path between the transmitter and load sockets is part of the main transmission line. This means that there will be a potential difference between the outsides of the two sockets and a corresponding potential gradient along the earth-plane. It follows that the lower voltage-sampling components and the Faraday shield should be either; star-wired to the detector socket ground; or connected along a line of equipotential (i.e., a line on the ground-plane drawn at an approximate right-angle to the through-line).

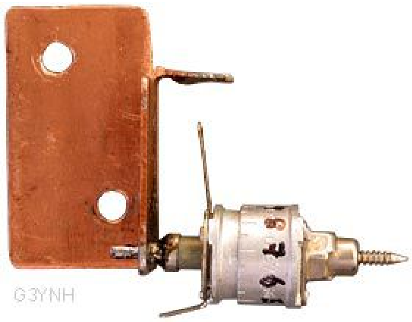
Socket ground connections of extremely low resistance are essential for accuracy. This can be achieved by fitting a copper sub-chassis to the inside of the box, and by using 4-bolt silver-plated BNC sockets with the flanges mounted on the inside.



Due to its high resistivity, Al-Zn die-casting alloy is not a material of choice for RF transmission lines. Low-impedance ground connections for the port sockets are obtained by means of a copper sub-chassis. Mounting the socket flanges on the inside ensures direct contact with the copper (rather than via the retaining screws). The 0.9 mm thick soft-copper sheet used was obtained by sawing open and flattening an offcut of ¾" (22 mm OD) water pipe.

The lid of the box is fitted with an inspection window to allow the scale marks on C_{1a} to be read. Clear polystyrene, acrylic or polycarbonate is suitable. Fibre washers are fitted under the retaining nuts to avoid cracking. Because the screws must not be done up too tightly, they are prevented from shaking loose by applying a drop of shellac varnish[7]. Shellac can be softened by applying alcohol if the threads need to be released; but note that stressed acrylic will shatter if wetted with alcohol. The inspection aperture is 12 mm × 16.5 mm. It was originally planned that a fine wire mesh would be fitted behind the window, to prevent the balance from being disturbed by fingers or objects placed on the outside; but this was found to be unnecessary.

7 Also known as 'button polish' or 'knotting'. Not suitable for vegetarians - may contain beetles.



The voltage-ratio adjustment capacitor C_{1a} is mounted on a copper bracket. This bracket also provides the grounding point for all of the lower voltage-sampling network components and the Faraday shield, giving a passable approximation to star-wiring.

Parts List

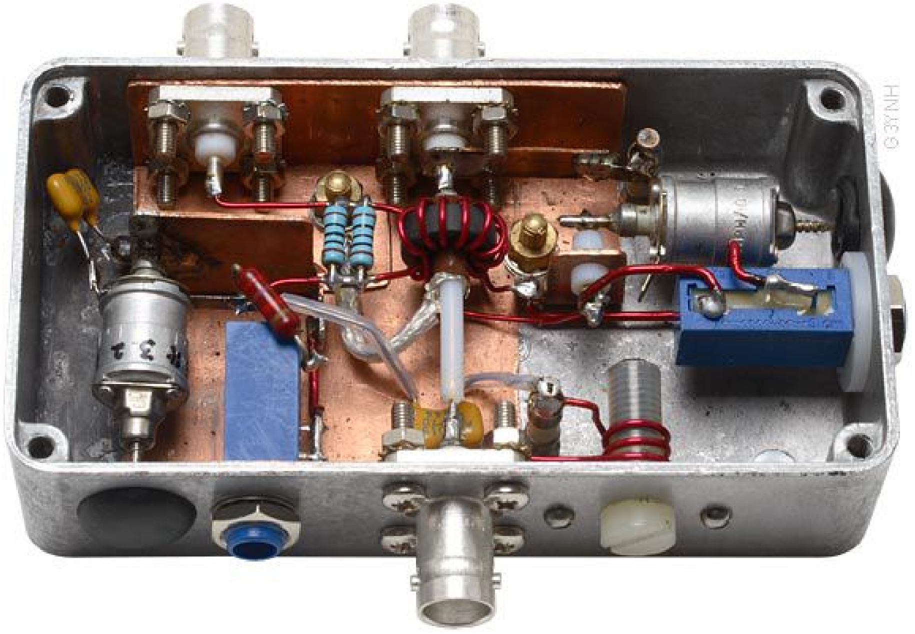
(neglecting fasteners and sundries)

Part:	**Description:**	Source or RS stock code.
Transformer core	Fair-Rite 5961000301 (Amidon FT50-61), ½" OD, A_L = 69 ±17 nH/turn²	Amidon Sycom
Through-line & Faraday shield	Uniradio M108, 50 Ω, Ag-plated Cu conductors, PTFE dielectric, FEP coat, 4.5 mm OD, 94 pF/m. (or similar).	
L_2 former	7mm dia., polystyrene, with M6 dust-iron slug.	
C_{1a}	Philips 3 pF - 30 pF beehive trimmer 5910-99-0167006 MPH/YL (modified, see text).	NOS [b]
C_{1b}	39 pF Ag-mica.	RS 495-694
C_2	4.7 pF, 500 V, Ag-mica.	RS 495-616
C_n	Philips 2 pF - 8 pF beehive trimmer.	NOS [b]
R_{ia} , R_{ib}	100 Ω, 0.5 W, metal film.	RS 149-644
R_{va}	Vishay ¾" Cermet trimpot, 20 turn, type 43p, 500Ω	RS 162-192
R_{vb}	2 kΩ - 3 kΩ (select on test), carbon or metal film [a].	
R_n	Vishay ¾" Cermet trimpot, 20 turn, type 43p, 50 Ω	RS 162-653
Trimpot mounts	Bourns H-38P panel mounting adapter for ¾" trimpots.	RS 107-6878
Box	Eddystone 7134p. 111 × 60 × 30 mm OD. (or similar).	RS 343-9524
Sockets	BNC 50 Ω, Ag-plated, PTFE insulated, 4-hole flange.	

Notes

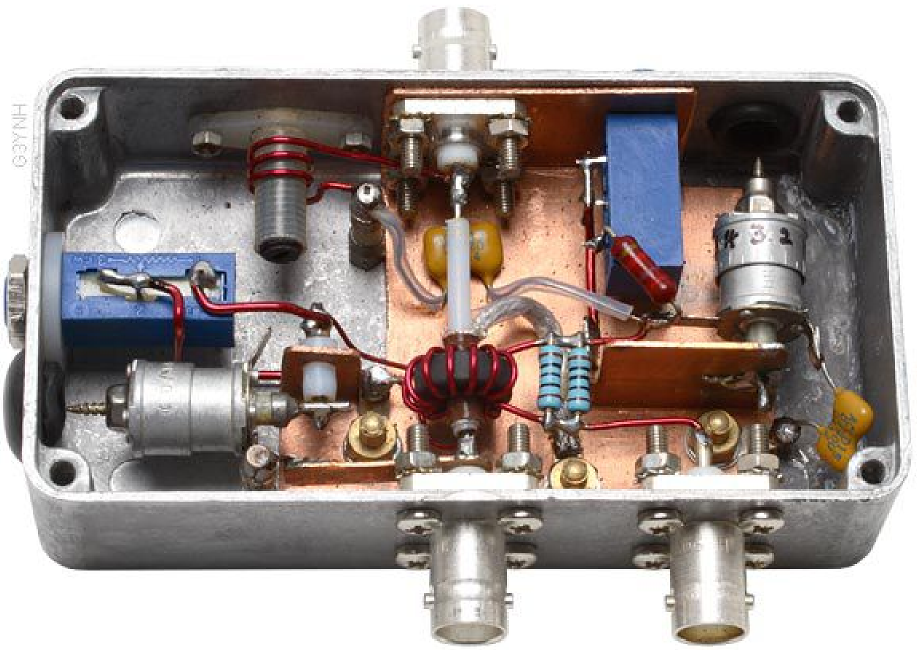
a) Beware the guilty secret that often lies beneath the smooth exterior. Perform the scrape test! If helical resistors are all that is available, there may be some advantage in using 3 or 4 resistors in parallel.

b) New Old Stock. Bought from a stall at a radio rally. Can also be salvaged from old radio equipment.

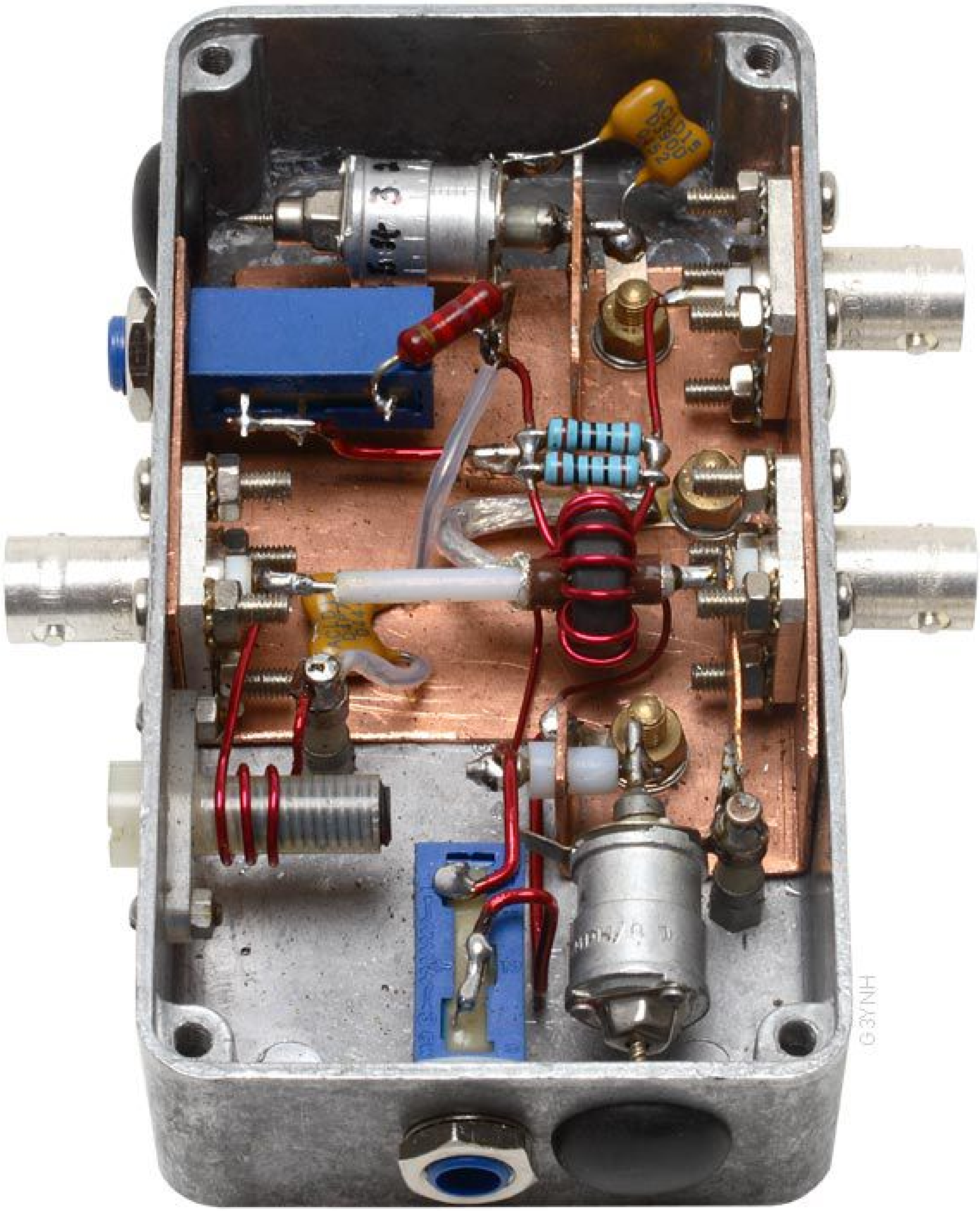


⅜" PVC blind-grommets are used to plug the 9.5 mm diameter capacitor access holes. A short M6 (0BA) Nylon set-screw discourages tampering with the core of L_2. A unit destined to be released into the wild should, in any case, bear a legend warning of the consequences of improper adjustment.

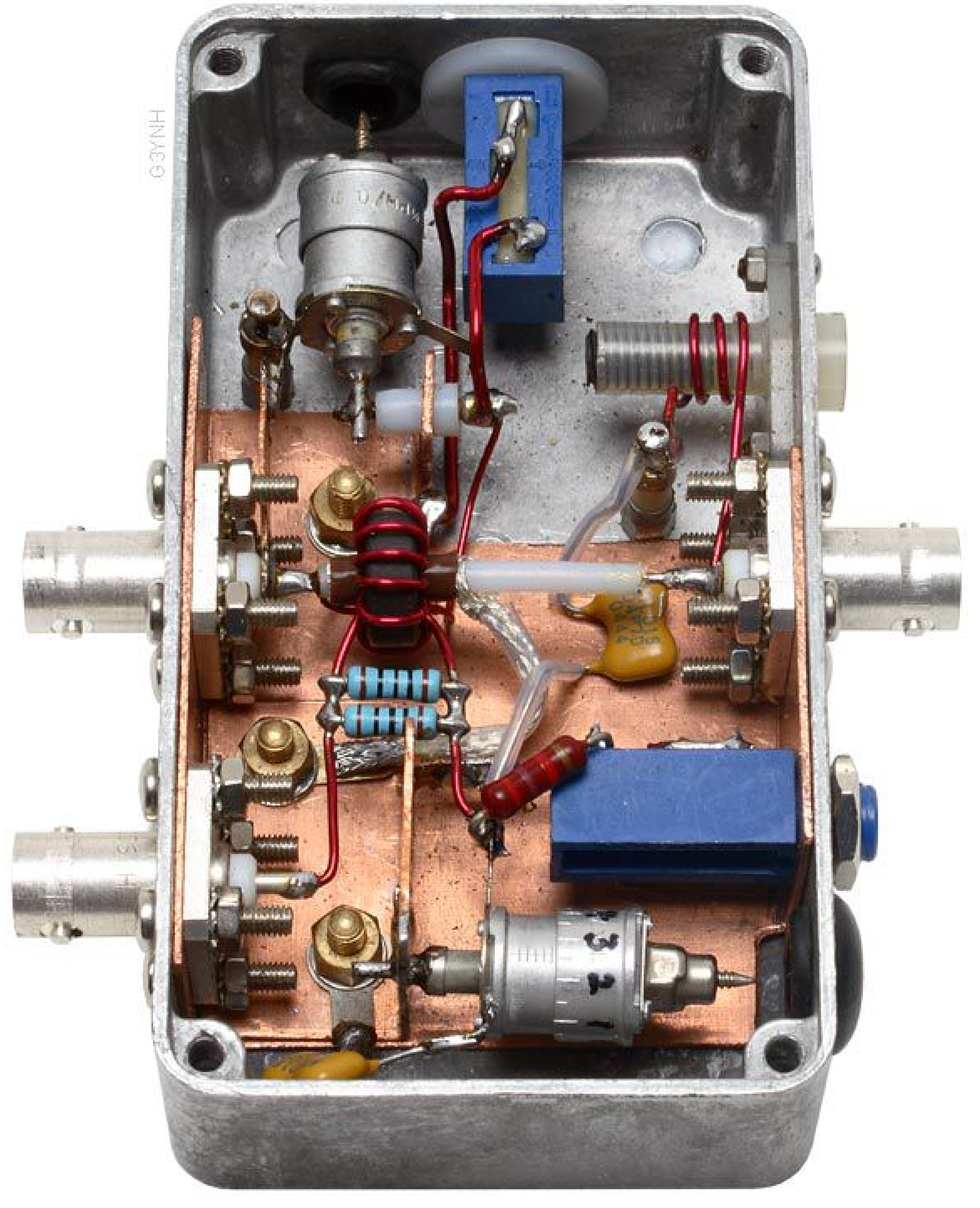
Molest not ye adjustments unless you knowest full well what you art doing.



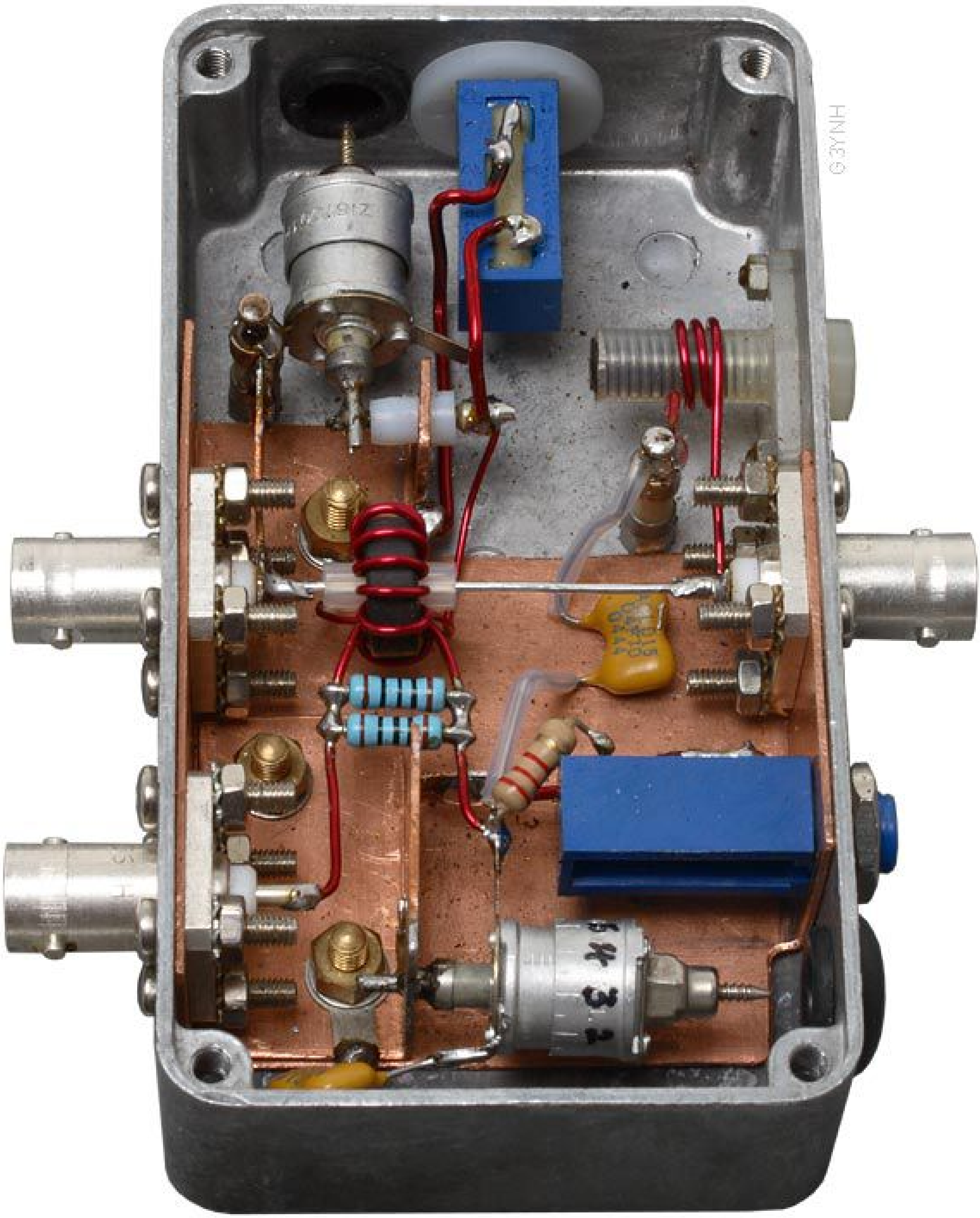
The polystyrene coil former for L_2 is mounted horizontally. This maximises the adjustment range by allowing the slug to be screwed well away from the coil. An 8 mm ID Nylon washer spaces the neutralisation pot. (R_n) back from the wall of the box. This is done merely to reduce the external protrusion. Note that L_2 is positioned so that its slug cannot crash into the pot..



Precision bridge with transformer Faraday shield fitted
Neutralisation pot. and capacitor in foreground. R_{vb} is 2.7 kΩ.



Precision bridge with transformer Faraday shield fitted
Lower VS capacitor C_1 in foreground.



Precision bridge without Faraday shield.
During soldering; the polythene insulator is moved away from the point being soldered and the line is clamped with pliers or forceps to divert the heat. To keep the amount of heat required to a minimum, unplug the test leads and the dummy load. A heat shunt is also required to protect the polystyrene former of L_2 during soldering. Note that R_{vb} is 2.2 kΩ.

3. Setup and calibration

Equipment required

● A laboratory signal generator covering at least 1.6 MHz to 30 MHz; output level ca. 0 dBm (224 mV RMS). Digital readout is desirable, but a VFO-type generator can be used with a digital frequency meter (DFM).

● A continuous coverage HF communications receiver with digital frequency readout; sensitivity ca. 100 nV for 6 dB s/n in 3 kHz bandwidth; with Fast AGC facility (the author used a Kenwood TS-930s).

● A 50 Ω ±1% coaxial load resistor, rated to several GHz (the reference resistance, R_{0cal})

● A resistance meter or multimeter, in a good state of calibration, preferably 4½-digit (e.g. Fluke 8060A).

● An adapter, with silver-plated conductors, for connecting the resistance meter to BNC plugs. A standard BNC to 4 mm adapter with 19 mm pin spacing fits most multimeters.

● A shorting plug, made from a silver-plated BNC plug, for zeroing the meter prior to measuring the reference resistance.

● A 10 cm BNC to BNC patch cable and a BNC T-piece; all preferably silver-plated.

● An in-line DC blocking capacitor[8].

● 2× common-mode chokes[9].

● Double or triple shielded 50 Ω test leads, with silver-plated connectors.

● Trimming tools and heat-shunt pliers (see below). A watchmaker's eyeglass.

● A small 5 kΩ 270° carbon or cermet preset pot.

● A selection of 0.25 W or 0.5 W (or thereabouts) carbon-film resistors covering the range 2.0, 2.2, 2.4, 2.7 and 3.0 kΩ, **OR** a selection of metal-film resistors (for connection in parallel in sets of 3 of the same value) covering the range 6.2, 6.8, 7.5, 8.2 and 9.1 kΩ.

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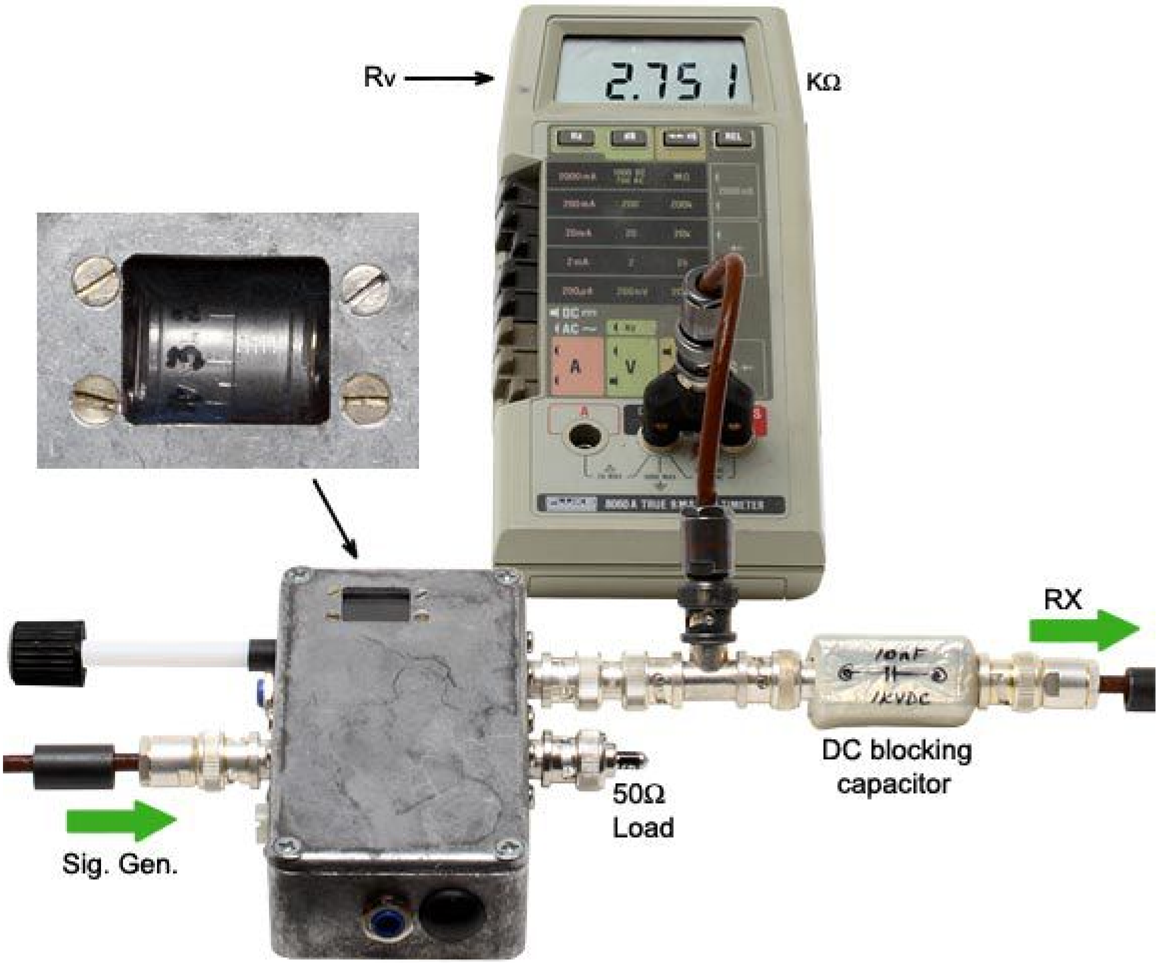
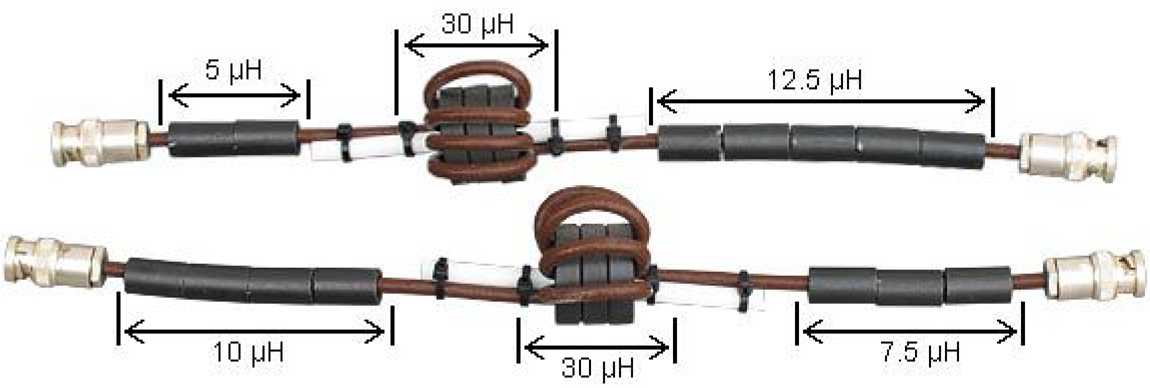
8 See **Eval. & Opt.** section 6.
9 See **Eval. & Opt.** section 7.



Test setup: R_v is measured continuously via a T-piece in the detector line. A DC blocking capacitor is needed when the input to the radio receiver is a DC short-circuit. The capacitance scale is read through a small polystyrene window using a watchmaker's eyeglass. Connections to the generator and the receiver are via common-mode chokes (see below).

The Fluke 8060A produces interfering signals at 16 MHz and 3.2 MHz. It can be switched off while the bridge is being balanced at those frequencies. Other meters will have their own idiosyncrasies.



Common-mode chokes used in the signal generator and receiver lines. The cable is URM108. The multi-section asymmetric construction prevents the self-resonance of any section from producing an overall low common-mode impedance.

Tools

1) Artery forceps (ratchet-locking pliers). Used to shunt heat away from low-melting-point plastics during soldering. A small pair of pliers with a rubber band around the handles is an alternative.

2) Improvised tool for adjusting beehive trimmers. A 2 mm diameter hole is drilled into the end of a 6 mm diameter plastic rod. A Neoprene Hellerman sleeve grips the nut on the trimmer. The addition of a small control knob improves the ergonomics. If the trimmer is stiff, remove the top and apply a minute quantity of MbS_2 grease to the ceramic insulator inside. DO NOT apply grease to the screw thread (it will increase the ESR if you do).

3) Original tool for early-pattern beehive trimmers. Made from SRBF (Whale Tufnol) with brass insert. Excessive metal content causes electrical disturbances when the outer electrode of the capacitor is not earthed. Not recommended.

4) Nylon trimming tool for hex. and slot-socket tuning slugs. Also used for adjusting 43p-style trimpots.

Receiver input cable used after the main CM choke from the detector port. Triple-shielded Belden 9880 with silver-plated connectors and adapters. The four Mn-Zn Ferrite beads give an additional 30 µH of choking inductance at the receiver input. Immunity from shield pickup requires that the PL259 connector and elbow at the receiver socket are done-up tightly.

3a. Optimising the test setup

The balance point of the bridge can be determined with extreme accuracy when using a sensitive radio receiver as the detector. The operator can however be fooled if the receiver has any sensitivity to common-mode signals. If the generator can be picked-up by a route other than through the detector port; the apparent balance point will be skewed because the null will occur, not at true balance, but at the point at which the signal from the bridge is of the correct magnitude and phase to cancel the spurious signal. There are three main causes of this spurious sensitivity:

● **Poor shielding** of the antenna input cable. This problem can be solved by using braided and taped cable (such as Belden 9880) for the line to the receiver (see above).

● **High-resistance shield connections** at the antenna socket and at joints in the cable. This problem can be solved by using silver-plated connectors throughout. If however, the receiver has that most execrable of input connectors, the cheap nickel-plated SO239, there is much to be said for replacing it with a 4-hole silver-plated N-type connector, bolted to the chassis with extreme prejudice. It takes only a few milli-Ohms at the antenna socket ground to let the receiver pick-up broadcast stations on the outside of a 1.5 m length of coaxial cable.

● **Earth-loop currents** due to electrical continuity between the RF terminals and the mains earth of most items of test equipment. This problem can be solved by installing a common-mode choke (unun) in the line from the generator to the bridge and in the line from the bridge to the receiver, or preferably both (see above). Multi-stage asymmetric construction of the CM chokes helps to prevent spurious effects due to self-resonances.

Detecting the balance point

When the bridge is balanced, the signal at the detector port (when using a generator output of around 0 dBm) will fall below the noise level of a very good communications receiver. Also, during the process of commissioning; when the bridge begins to get close to its final working condition; it will, to all intents and purposes, never go out of balance when the operating frequency is changed. Using a VFO-type signal generator, it then becomes practically impossible to hear the detector signal while sweeping through, unless the bridge is deliberately thrown out of balance first. When testing with the lid off, the balance can be thrown temporarily by inserting a finger into the works, but this cannot be done once the lid is on. This is a nuisance unless the frequencies of the generator and receiver can be set accurately, because the process of calibrating the bridge is not assisted by the need to keep disturbing the adjustments. Hence both generator and receiver should have reasonably accurate digital frequency readouts. Using USB mode, the receiver is set about 800 Hz lower than the generator at each spot-test frequency.

3b. Commissioning procedure

Before the trimpots are connected to the circuit, they should be set to the middle of travel with the aid of a resistance meter. A small 5 kΩ 270° preset pot (on the shortest possible wires) can be connected in place of R_{Vb} in order to determine the best value for the intended fixed resistor. This determination can be done to good-enough accuracy with the lid off.

Start with about 4-turns on L_2. When the temporary pot. is replaced by a fixed resistor, this can be reduced to 3 turns, provided that the resistor is of low inductance. Carbon-film resistors tend to be less inductive than metal-film resistors, for reasons hidden beneath the paint. There is an issue that CF resistors can be expected to have inferior long-term stability compared to MF, but the ones used by the author had been kept in an unheated store-room for more than 20 years and were still well within spec.. Some old carbon-slug (carbon composition) resistors tested however were all well out of spec., and therefore highly unsuitable for use in accurate instruments (and also for just about any other conceivable purpose). If helical-cut MF resistors must be used, there may be an advantage in making the required resistance by connecting 3 or 4 identical resistors in parallel.

Having attached the reference load resistor to the load port and connected the test equipment; the alignment procedure is as follows:

- At 2 MHz: balance the bridge using C_1 and R_V.
- At 28MHz: balance the bridge using L_2 and C_n.
- At 18MHz: minimise the detector signal by adjusting R_n.
- Repeat the above three steps in the given order until no further improvement can be obtained.

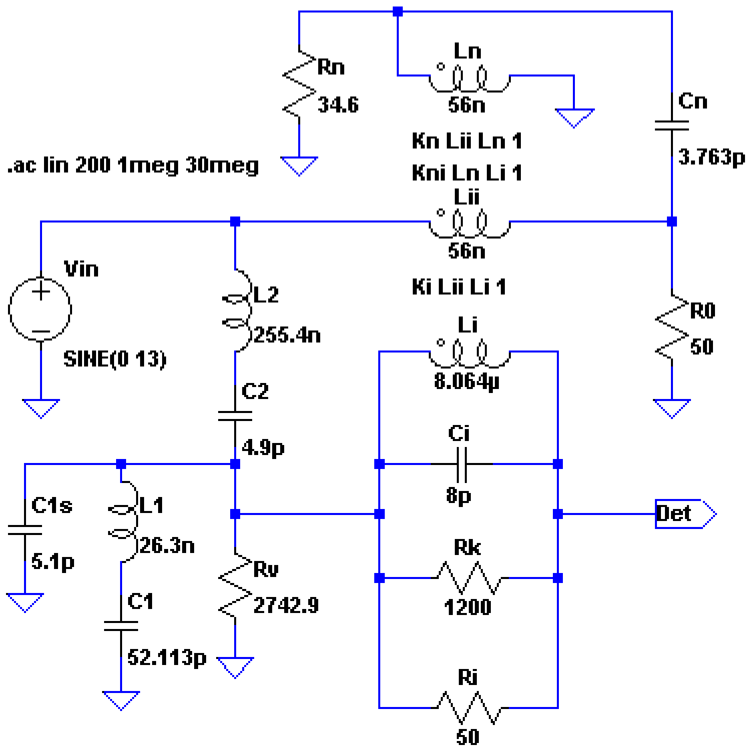
After the first try at alignment, it is worthwhile to disconnect one end of R_n and check that it is somewhere in the region of 20 Ω to 25 Ω. A low setting for R_n, particularly if it is not considerably greater than 4.2 Ω (R_i / N = 50 / 12), is strongly indicative of a defect in some other part of the circuit.

Once a fixed resistor is in place for R_{Vb}, the lid of the box should be screwed down tightly and the alignment procedure repeated. It will be found that the inductance of L_2 needs to be reduced when the lid is put on, so it is a good idea to arrange things so that the slug is fairly well into the coil after alignment with the lid off. The number of turns and the turn-spacing can be adjusted for that purpose.

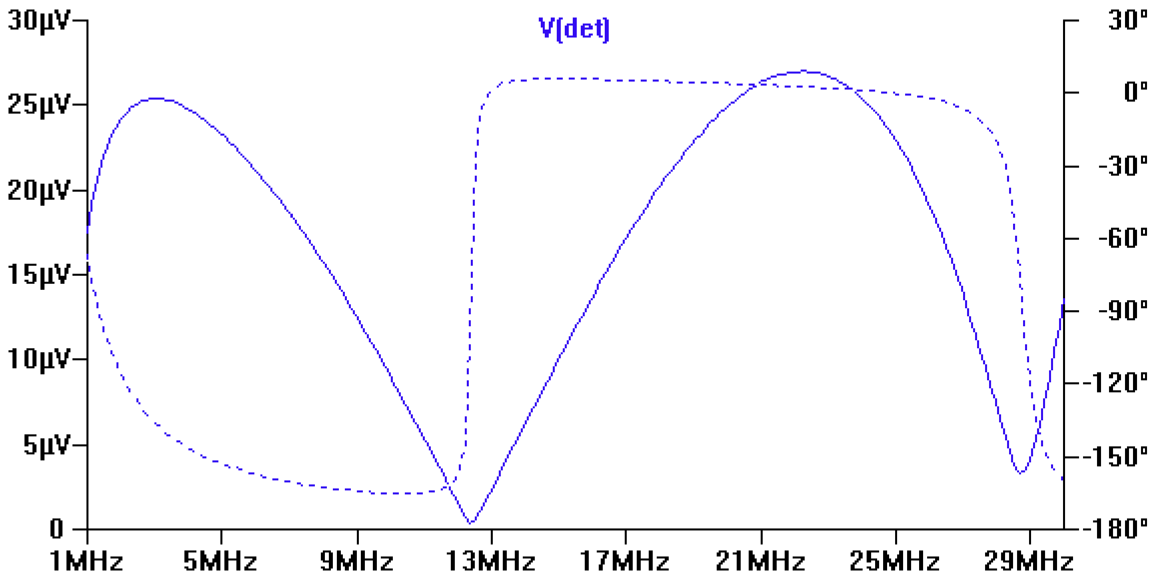
The bridge is then evaluated to see if it has achieved its performance targets; and if not, the circuit is investigated for faults and the whole process is repeated

3c. SPICE simulation

For those who would like to experiment with the circuit in simulation, the circuit file **refbrg_fs.asc** can be opened using the LTspice simulator[10]. LTspice will convert the schematic into a pspice compatible netlist for its own or for other simulators. C_1, R_v, L_2, C_n and R_n can be adjusted according to the commissioning procedure described above to minimise the detector output over the chosen frequency range.



The parasitic elements; C_{1s} (strays from summing point to ground); L_1 (inductance of lower voltage-sampling arm); and C_i (equivalent secondary parallel capacitance), have been put into the circuit as separate components. Transformer leakage inductance is not included. Note that this simulation will not lead to a realistic value for R_n because the model does not account for dispersion effects in the type 61 ferrite material used for the transformer core. The value placed against L_{ii} and L_n is the core inductance factor (A_L). $L_i = A_L N^2$ (and N=12). Transformer coupling coefficients K_n, K_{ni} and K_i have all been set to 1.



10 Available free from the Linear Technology website: http://www.linear.com/designtools/software/
LTspice is a Windows program, but runs perfectly under Wine.

4. Theory of the evaluation process

During the calibration process, the low-frequency balance-point is set by connecting a reference resistor to the load port and adjusting C_{1a} and R_{Va}. C_{1a} adjusts the in-phase (resistance) balance, and R_{Va} adjusts the quadrature (reactance) balance. The lower voltage-sampling capacitance (C_1) is chosen as the variable parameter because this makes the in-phase and quadrature balance adjustments non-interactive. A further advantage of adjusting C_1 rather than C_2 is that one end of the trimmer capacitor is grounded.

Provided that the high-frequency balance errors are not too great, C_{1a} and R_{Va} can be used to balance the bridge exactly at any frequency in the working range. Small deviations from the low-frequency calibration values can then be used to compute bridge performance figures. As mentioned previously; $R_V = R_{Va} + R_{Vb}$ is measured via the detector port, and relative capacitance values are obtained by peering through a small inspection window with an eyeglass and converting the logging-scale readings by means of a regression formula.

When a bridge is corrected for resistance-balance errors, the inductance of the lower voltage sampling arm is cancelled out. For this reason, the compensation coil L_2 does not need to be included in the balance equation because it exists only to make C_1 look like a pure capacitance. Similarly, when the bridge is corrected for reactance-balance errors, we do not need to include the transformer self-capacitance in the balance equation, because the purpose of the neutralisation network is to make it appear that the reactance of the current transformer secondary winding is purely inductive. On that basis (using the derivation given in the preceding article[11]) an accurate expression for the balance condition for a doubly-corrected bridge is:

$$\frac{C_1}{C_2} + 1 + \frac{\mathbf{j}X_{C2}}{R_V} - \frac{1}{N} = (1 + C_x / C_2)\, N\, \mathbf{Z_0} \left[\frac{1}{k'\, R_i} - \frac{\mathbf{j}}{X_{Li}} \right] \qquad (\mathbf{1})$$

where the values inserted for C_1 and C_2 must include strays; $X_{C2} = -1 / (2\pi f\, C_2)$, C_x is the stray capacitance from the through-line to the detector port (C_x is negligible for a well laid-out bridge with a Faraday shield); N is the number of turns in the transformer secondary; $\mathbf{Z}_0$ is the impedance connected to the load port (defined as $\mathbf{Z}_0 = R_0 + \mathbf{j}X_0$); k' is the transformer efficiency factor (about 0.96 for the transformer used[12]); R_i is the transformer secondary load resistance; and X_{Li} is the inductive reactance of the transformer secondary winding.

One difference between equation **(1)** and the original derivation is that we have substituted $\mathbf{Z}_0$ in place of R_0. This has been done to allow us to analyse the the case when there is some residual load reactance when the bridge is at balance. Now, multiplying $\mathbf{Z}_0 = R_0 + \mathbf{j}X_0$ into the square bracket, we get the explicit balance condition for an arbitrary load:

$$\frac{C_1}{C_2} + 1 + \frac{\mathbf{j}X_{C2}}{R_V} - \frac{1}{N} = (1 + C_x / C_2)\, N \left[\frac{R_0}{k'\, R_i} + \frac{\mathbf{j}X_0}{k'\, R_i} - \frac{\mathbf{j}R_0}{X_{Li}} + \frac{X_0}{X_{Li}} \right] \qquad (\mathbf{2})$$

11 **Eval. & Opt.** Section 19.

12 **Current transformer efficiency factor.** D W Knight. http://g3ynh.info/zdocs/bridges/

4a. Resistance balance error

The resistance balance condition is obtained by dropping all of the imaginary terms from equation **(2)**:

$$\frac{C_1}{C_2} + 1 - \frac{1}{N} = (1 + C_x / C_2)\, N \left[\frac{R_0}{k' R_i} + \frac{X_0}{X_{Li}} \right] \qquad (3)$$

This expression can be rearranged in favour of R_0

$$R_0 = \frac{k' R_i}{(1 + C_x / C_2)\, N} \left[\frac{C_1}{C_2} + 1 - \frac{1}{N} - (1 + C_x / C_2)\, N \frac{X_0}{X_{Li}} \right] \qquad (4)$$

The evaluation method is a type of perturbation analysis. The logic of it is that, if the capacitance C_1 has to be changed to rebalance the bridge on moving away from the calibration frequency, then the same effect can be had by leaving the capacitor alone and making a suitably proportioned change to the load resistance. Hence the deviation of the capacitance from its calibration setting is a measure of the resistance error that will occur if the calibration is left untouched. The amount by which R_0 must change for a given change in C_1 is given by differentiating R_0 with respect to C_1, i.e.:

$$\frac{\partial R_0}{\partial C_1} = \frac{k' R_i}{(C_2 + C_x)\, N} \qquad \text{[Ohms per Farad]} \qquad (5)$$

Hence, if δR_0 represents a change in R_0, and δC_1 represents a change in C_1, we can determine the relationship that exists between these changes by equating their ratio to the derivative **(5)**, i.e.:

$$\frac{\delta R_0}{\delta C_1} = \frac{\partial R_0}{\partial C_1} = \frac{k' R_i}{(C_2 + C_x)\, N}$$

For reasons of mathematical consistency, changes or errors are always defined as "obs. minus calc.", i.e., the observed or actual value minus the expected or calculated value. Hence the shift in R_0 is defined as:

$$\delta R_0 = R_0 - R_{0cal}$$

where R_0 is the actual load resistance and R_{0cal} is the reference load resistance.
The shift in C_1 is defined as:

$$\delta C_1 = C_1 - C_{1cal}$$

where C_{1cal} is the calibration setting and C_1 is the setting required when the load resistance is R_0.

To get the correct sign of the resistance error when the change in C_1 is due to a defect of the bridge however requires a little careful thought. The derivative **(5)** is positive, which means that an increase in R_0 mandates an increase in C_1. If however, C_1 should increase spontaneously due to a defect in the system, then R_0 must be *reduced* to restore the capacitor to its calibration setting. Hence we translate spontaneous changes in C_1 into resistance balance errors by taking the *negative* of the derivative. Thus, if δR_0 represents the predicted error in R_0:

$$\frac{\delta R_0}{C_1 - C_{1cal}} = \frac{-\partial R_0}{\partial C_1} = \frac{-k' R_i}{(C_2 + C_x)\,N}$$

Hence, resistance balance error is given by:

$$\delta R_0 = (C_{1cal} - C_1)\frac{\partial R_0}{\partial C_1} = \frac{(C_{1cal} - C_1)\,k' R_i}{(C_2 + C_x)\,N} \qquad (6)$$

Where the shift in C_1 has been changed to 'cal.-obs.' to get rid of the minus sign, i.e., $C_1 - C_{1cal} = -(C_{1cal} - C_1)$. Notice that **(6)** does not require that we have to know C_1. We only need to know the amount by which it has changed.

The parameters k', R_i, and $C_2 + C_x$, all remain substantially constant over the operating frequency range; which is another way of saying that the derivative in this case can be assumed to be a constant. Hence the percentage deviation of C_1 away from its calibration setting translates directly into a resistance error expressed as a percentage. Hence, if we do happen to have a rough estimate for C_1 (including strays), we can gain a fair idea of how good the bridge will be before subjecting the shift measurements to a formal analysis. For the bridge under discussion, the circuit parameters were chosen to make $C_1 = C_{1a} + C_{1b}$ + strays to be about 57 pF, and the beehive trimmer used was found to have an adjustment gradient of -3.32 pF/turn. This means that an adjustment of 0.066 pF (1/50 of a turn) corresponds to a shift in the resistance balance of about 0.12%. Hence, if the bridge is to be accurate to better than ±0.12%, the trimmer must be shifted from its calibration position by no more than ±1/50th of a turn over the 1.6 MHz to 30 MHz range. With a practised eye, the trimmer position can be logged to 1/200th of a turn, with an uncertainty of ±1/200th of a turn. Hence the resolution for the in-phase performance evaluation process lies at about 0.029%. Typically, during the various tests carried out by the author, the shift was about ±1/100th of a turn, i.e., about ±0.058%.

We can also get an idea of how well the commissioning process is going by estimating the derivative explicitly. For the bridge described here: k' = 0.96 , R_i = 50 Ω , C_2 = 4.9 pF (including strays), C_x = 0 when a Faraday shield is used, and N = 12 . This gives:

$\partial R_0 / \partial C_1 = 0.8163$ Ω/pF

A shift of 1/200th of a turn corresponds to a shift of 0.0166 pF, which corresponds to a deviation of the resistance balance by 0.8163 × 0.0166 = 0.0135 Ω, this being 0.027% of 50 Ω.

4b. Reactance balance error

The reactance balance condition is obtained by dropping all of the real terms from equation **(2)** and cancelling-out the imaginary operators (i.e., **j**) on both sides:

$$\frac{X_{C2}}{R_V} = (1 + C_x / C_2)\, N \left[\frac{X_0}{k'\, R_i} - \frac{R_0}{XL_i} \right] \qquad (7)$$

Using the substitutions; $X_{C2} = -1/(2\pi f\, C_2)$ and $X_{Li} = 2\pi f\, L_i$ gives:

$$\frac{-1}{2\pi f\, C_2 R_V} = (1 + C_x / C_2)\, N \left[\frac{X_0}{k'\, R_i} - \frac{R_0}{2\pi f\, L_i} \right]$$

Multiplying top and bottom of the first term in square brackets by 2πf, cancelling through and multiplying both sides by -1 gives:

$$\frac{1}{C_2 R_V} = (1 + C_x / C_2)\, N \left[\frac{R_0}{L_i} - \frac{2\pi f\, X_0}{k'\, R_i} \right] \qquad (8)$$

If the bridge is perfect, X_0 is always 0, and so X_0 itself is a measure of the reactance error. Interestingly, if we treat X_0 as a capacitive reactance defined as $X_0 = -1/(2\pi f\, C_{0s})$, we get:

$$\frac{1}{C_2 R_V} = (1 + C_x / C_2)\, N \left[\frac{R_0}{L_i} + \frac{1}{C_{0s}\, k'\, R_i} \right] \qquad (9)$$

This (recalling that $\mathbf{Z_0} = R_0 + \mathbf{j}X_0$) shows that reactance error can be represented as a hypothetical frequency-independent capacitance (C_{0s}) (which can be positive or negative) *in series* with the load impedance. Here however, we will prefer series reactance, because it can be most readily converted into phase (using $\tan\varphi = X_0 / R_0$). Rearranging **(8)** in favour of X_0 gives:

$$X_0 = \frac{k'\, R_i}{2\pi f} \left[\frac{R_0}{L_i} - \frac{1}{N\,(C_2 + C_x)\, R_V} \right] \qquad (10)$$

The relationship between reactance error (δX_0) and deviation of R_V from its low-frequency calibration setting is given by differentiation as before. In this case, the power of R_V is -1 , and if $y = x^{-1}$, then $dy/dx = -x^{-2}$. Hence:

$$\frac{\partial X_0}{\partial R_V} = \frac{k'\, R_i}{2\pi f\, N\,(C_2 + C_x)\, R_V^2} \qquad \text{[Dimensionless]} \qquad (11)$$

In this case, we define $\delta X_0 = X_0$ (because $X_{0cal} = 0$) and $\delta R_V = R_V - R_{Vcal}$, where R_V is the actual setting and R_{Vcal} is the calibration setting. Once again, we take the negative of the derivative in order to find an expression for the reactance error:

$$\frac{\delta X_0}{\delta R_V} = \frac{-\partial X_0}{\partial R_V} = \frac{-\,k'\,R_i}{2\pi f\,N\,(C_2 + C_x)\,R_V^2}$$

Hence, reactance balance error is given by:

$$\delta X_0 = (R_{Vcal} - R_V)\,\frac{\partial X_0}{\partial R_V} = \frac{(R_{Vcal} - R_V)\,k'\,R_i}{2\pi f\,N\,(C_2 + C_x)\,R_V^2} \qquad (12)$$

Here the derivative is frequency dependent in such a way that the shift in R_V corresponding to a given phase error is much larger at high frequencies than it is at low frequencies. The derivative is also proportional to the reciprocal of R_V^2, which means the error expression is only true for small changes in R_V; but since the shifts encountered in practice are <1%, this is not a problem.

The resistance R_V is found by measurement, being dependent on the secondary inductance L_i. For the Faraday shielded version of the bridge described here ($L_i = 8.06\ \mu H$) the calibration setting was about 2748 Ω. Strictly, we should use the actual value of R_V rather than the calibration value when calculating the derivative, but for the purpose of illustration it doesn't make much difference. Including the other values: $k' = 0.96$, $R_i = 50\ \Omega$, $C_2 = 4.9$ pF , $C_x = 0$, $N = 12$, we get:

$\partial X_0 / \partial R_V = 17205 / f$

It is the precision rather than the accuracy of a measurement that must be taken into account when determining differences. The Fluke 8060A gives a readout to the nearest 1 Ω on the 20 kΩ range, giving a precision of ±0.5 Ω for a single reading and an RMS uncertainty of ±0.7 Ω for a difference. Hence using a 4½ digit meter gives the system a phenomenal RMS resolution for the measurement of reactance or phase error; as shown in the table below.

		RMS measurement resolution		Performance Criterion
f	$\partial X_0/\partial R_V$	δX_0 / Ω	$\delta\varphi$	Max δR_V for ±<0.1° $\delta\varphi$
/ MHz	= 17205 / f	= 0.7 $\partial X_0/\partial R_V$	=Arctan(δX_0/50)	= 50 f Tan(0.1°) / 17205
1.6	0.010753	±0.007527	±0.008625°	± 8.1 Ω
2	0.008602	±0.006022	±0.006900°	± 10.1 Ω
3	0.005735	±0.004014	±0.004600°	± 15.2 Ω
5	0.003441	±0.002409	±0.002760°	± 25.4 Ω
8	0.002151	±0.001505	±0.001725°	± 40.6 Ω
12	0.001434	±0.001004	±0.001151°	± 60.9 Ω
17	0.001012	±0.000708	±0.000812°	± 86.2 Ω
23	0.000748	±0.000524	±0.000600°	± 116.7 Ω
30	0.000573	±0.000401	±0.000460°	± 152.2 Ω

The RMS phase resolution is better than ±9 milli-degrees at 1.6 MHz, reaching $\pm 10^{-3}$ degrees at 13.8 MHz. Hence we need not concern ourselves with the issue of whether we can get better information using impedance analysers or vector voltmeters.

For guidance while testing the bridge, it is useful to produce the kind of information shown in the right-most column. The figures tell us the maximum deviation from R_{Vcal} that can be allowed if the bridge is to have an overall phase accuracy of better than ±0.1°. The calculation formula is derived as follows:

$$\delta X_0 = -\delta R_V \, \partial X_0/\partial R_V$$

and

$$\mathrm{Tan}(\delta\varphi) = \delta X_0 / R_0$$

Hence

$$R_0 \, \mathrm{Tan}(\delta\varphi) = -\delta R_V \, \partial X_0/\partial R_V$$

i.e.,

$\delta R_V = -R_0 \, \mathrm{Tan}(\delta\varphi) / (\partial X_0/\partial R_V)$	**13**

For the example bridge, $\partial X_0/\partial R_V = 17205 / f$, and $R_0 = 50\ \Omega$. Hence, ignoring the sign because we are only interested in the magnitude of the deviation:

$$|\delta R_V| = 50 \, f \, \mathrm{Tan}(\delta\varphi) / 17205$$

If we put in a phase performance criterion as $\delta\varphi$, the formula returns a maximum allowable resistance deviation as $|\delta R_V|$. The number 17205 applies of course, only to the example. The correct value should be calculated from the parameters of the actual bridge under test.

The formula is practically linear for small phase errors. Hence it is possible to estimate the phase error at a particular frequency as a proportion of the ±0.1° criterion. For example, referring to the table above; If the bridge balances when R_V is shifted away from R_{Vcal} by 30 Ω at 12 MHz, the phase error at 12 MHz is within ±0.05°.

If the guidance given here is followed, and provided that the temperature of the transformer core remains reasonably constant during the evaluation procedure, the bridge should have a phase performance no worse than ±0.03° at any frequency in the 1.6 to 30 MHz range (although it may take some practice with the alignment procedure before this is achieved). If there are any peculiar kinks or bumps in the phase response, these are most likely to be due to resonances in the common-mode chokes. Hence, if the bridge is free from obvious faults but refuses to meet its performance targets, it is worth investigating and possibly redesigning the chokes.

5. Evaluation procedure and final alignment

The evaluation procedure is based on the application of two formulae; equations **(6)** and **(12)**. The author's preference is to use the Open Office spreadsheet program[13] for the calculations, and test results given below are consequently in the form of open-document spreadsheet (.ods) files.

The notionally separate evaluations for resistance and reactance balance can be combined to produce results expressed as magnitude and phase error. The magnitude error can also be expressed relative to the design load resistance ($R_{0d} = 50\ \Omega$ usually) rather than the actual reference load resistance (which is probably not exactly $50\ \Omega$). By adjusting (in the spreadsheet) the initially used calibration setting of the capacitor scale, a setting can be found that distributes the errors evenly above and below the design load resistance. Final alignment (equivalent to having used a reference resistance that was exactly on target) can then be obtained by applying this setting to the bridge

A further refinement to the evaluation procedure is to use the required input parameters to calculate the non-essential parameters L_i and C_1. Realistic values for these quantities show that the circuit is free from physical defects.

The fixed input parameters used in the full analysis are:

R_{0cal}	Ref. load resistance (preferably within 1% of R_{0d}).	Measured accurately
σ_{R0}	Uncertainty in the measurement of R_{0cal} .	From resistance meter manual.
R_{0d}	Design load resistance	Usually 50 Ω
k'	Transformer transfer-efficiency factor.	about 0.96 ±1%
R_i	Secondary load = R_{ia} // R_{ib}	preferably measured
N	Number of secondary turns on the transformer.	
C_2	Upper voltage sampling capacitance, including strays.	
C_x	Strays from through-line to detector port.*	= 0 when Faraday shield is used
x_{cal}	C_{ia} scale reading at 2 MHz	Starting value. Optimised later
σ_x	Uncertainty in the reading of the capacitor scale.	0.005 of a turn
R_{Vcal}	R_v reading at 2MHz	Starting value. Optimised later.

* The inscrutable nature of C_x would seem to mandate the use of a Faraday shield; but a sensible estimate will do (ca. 0.23 pF), and for those prepared to run tests on both shielded and unshielded versions of the bridge, it can be determined accurately.

Before the bridge is built, measurements of capacitor scale-reading (x) vs capacitance (C_{1a}) are fitted to a regression line (see box at the end of section **1**). The spreadsheet used for that calculation is pasted into a separate sheet of the bridge evaluation file. The fit returns two parameters, a and b, that are used to convert scale readings into capacitances according to the formula:

$C_{1a} = a + bx$

Notice that we do not need to know the actual value of $C_1 = C_{1a} + C_{1b} + \text{strays}$, because the analysis **(6)** requires only differences, i.e:

$-\delta C_1 = C_{1cal} - C_1 = C_{1acal} - C_{1a}$

13 Available free from http://www.openoffice.org/

strictly, we do net even need to know the fitting parameter a , but excessive elimination of redundancy makes it difficult to spot scale interpretation errors and other mistakes.

A datum for the evaluation is a set of three numbers (f , x , R_v). Measurements are made at a representative set of spot frequencies, with the readings more crowded at low frequencies than at high frequencies (i.e., very crudely, at approximately equal intervals on a logarithmic scale). The author used a set of 29 frequencies in the 1.6 MHz to 30 MHz range. It is easiest to start taking readings at 30 MHz and work downwards.

Fewer data are required at high frequencies because the evolution of the phase error is very smooth in that region. The reason is that the experimental resolution is extremely good at high frequencies and, apart from a very low-Q dispersion effect in the type 61 ferrite used in the transformer, there is nothing in the box that can resonate at less than 30 MHz. As mentioned before, if glitches are seen, they are almost certainly due to the external test setup.

The calculated phase errors will appear more chaotic at low frequencies. This (presuming that the transformer core temperature remains constant) is not due to imperfections in the bridge, but due to reduced resolution and increasing difficulty in finding the exact balance point. In the 1.6 MHz to 3 MHz region, the detector signal disappears below the receiver noise for about ±1 turn of the trimpot R_{Va} , meaning that the mid point has to be found by estimation. A setting error of ±2 Ω at 1.6 MHz looks like a phase error of about ±0.025°, so the apparent scatter is an artefact. The solution is to take more closely-spaced readings at low frequencies, so that the scatter can be averaged to find a good final value for R_{Vcal} .

5a. Magnitude and phase error. Final calibration

The resistance and reactance errors are increments to an impedance expressed in the normal series form:

$\mathbf{Z_0} = R_0 + \mathbf{j}X_0$

where:

$R_0 = R_{0cal} + \delta R_0$

and

$X_0 = \delta X_0$

because $X_{0cal} = 0$

The magnitude of the predicted load impedance for perfect balance (i.e, the load impedance including the errors) is:

$|\mathbf{Z_0}| = \sqrt{(R_0^2 + X_0^2)}$

where, strictly, X_0 is the error returned from the reactance balance evaluation (but setting it to zero makes very little difference if X_0 is small).

Hence the magnitude error is:

$\delta|\mathbf{Z_0}| = |\mathbf{Z_0}| - R_{0d}$

If the design load resistance R_{0d} is used as the comparison reference, the initial capacitor calibration setting x_{cal} can be tweaked to distribute the errors around R_{0d}, thereby determining the optimal final setting for the capacitor.

The phase angle of the predicted load impedance is given by:

$\varphi = \mathrm{Arctan}(X_0 / R_0)$

where, strictly, R_0 is the load resistance obtained from the resistance balance evaluation (but setting it to 50 Ω makes practically no difference). Since the target phase angle is zero,

$\delta\varphi = \varphi$

The value of R_{Vcal} in the spreadsheet can be tweaked to get the best distribution of errors above and below zero. This is the optimal final setting for R_V.

5b. Final accuracy and realistic performance claims

Since the experimental phase resolution is much better than the phase accuracy of the bridge over most of the frequency range, the final phase accuracy at the evaluation temperature is given by the maximum or minimum excursion of the phase error away from zero (i.e., $\pm|\delta\varphi|_{max}$). The objective, in calibration and in choice of the final value for R_{Vcal}, is to make the maximum positive and negative excursions about equal. At low frequencies, small excursions outside the high-frequency phase-performance limits can be discounted. These are artefacts and can be reduced by obtaining more practice at balancing the bridge.

The phase accuracy of the bridge is affected by temperature. This issue is investigated in section **6**.

Although the experimental phase resolution is very good; the experimental magnitude resolution is limited by the scale resolution of the cheap and cheerful modified beehive trimmer. There will also be uncertainty contributions due to the uncertainty in the measurement of the reference load resistance (R_{0cal}), and due to the uncertainty in making the final setting of the trimmer. Hence there will be a statistically significant difference between the experimentally-determined precision of the bridge (i.e., the maximum RMS magnitude runout obtained from the raw data) and the claimable magnitude accuracy.

Defining the true or claimable magnitude accuracy of the bridge as σ_Z, we can write an expression for this quantity on the basis that various errors that contribute to it are uncorrelated[14]; i.e., we add the various uncertainty contributions as though they are a set of orthogonal vectors. Thus:

$$\sigma_Z = \sqrt{[\ |\delta\mathbf{Z_0}|_{max}^2 + \delta_{Z,x}^2 + \delta_{Z,R0}^2 + \delta_{Z,xcal}^2\ ]} \quad \ldots\ldots\ldots\ldots\ \mathbf{(14)}$$

14 See **Data analysis**.

where:

$|\delta \mathbf{Z_0}|_{max}$ is the maximum RMS magnitude runout (the precision of the bridge);
$\delta Z_{,x}$ is the uncertainty contribution due to uncertainty in reading the capacitor scale;
$\delta Z_{,R0}$ is the uncertainty contribution due to uncertainty in the measurement of R_{0cal};
$\delta Z_{,xcal}$ is the uncertainty contribution due to uncertainty in the final capacitor setting.

For the three additional uncertainty contributions, we apply the rule: The uncertainty contribution due to a particular parameter (or variable) is equal to the rate of change of the desired quantity with respect to the parameter multiplied by the uncertainty in the parameter. Hence:

$$\delta Z_{,x} = (\partial|\mathbf{Z_0}| / \partial x)\ \sigma_x$$

$$\delta Z_{,R0} = (\partial|\mathbf{Z_0}| / \partial R_0)\ \sigma_{R0}$$

$$\delta Z_{,xcal} = (\partial|\mathbf{Z_0}| / \partial x)\ \sigma_x$$

Note that two of the uncertainty contributions in this case are the same, so we only have two derivatives to find. Moreover, we can simplify the problem enormously by making a very minor approximation (the point is to find an estimated standard deviation, we do not require large numbers of significant figures). We first note that $|\mathbf{Z_0}|=\sqrt{(R_0^2 + X_0^2)}$, and that by proper application of the 'function of a function' rule, we get:

$$\partial|\mathbf{Z_0}| / \partial R_0 = R_0 / [\ \sqrt{(R_0^2 + X_0^2)}\ ]$$

but this derivative is so close to unity that we might as well take it to be unity. Hence:

$$\partial|\mathbf{Z_0}| = \partial R_0$$

Now, feeding this back into our error contributions we get:

$$\delta_{Z,x} = (\partial R_0 / \partial x)\ \sigma_x$$

$$\delta_{Z,R0} = \sigma_{R0}$$

$$\delta_{Z,xcal} = (\partial R_0 / \partial x)\ \sigma_x$$

We already have an expression for R_0 in terms of C_1 in the form of equation **(4)**. We can also express C_1 as:

$$C_1 = C_{1a} + C_{1b} + C_{1s}$$

where C_{1s} is the contribution from strays. C_{1a} is given by the regression formula as:

$$C_{1a} = a + bx$$

Hence:

$$C_1 = a + bx + C_{1b} + C_{1s}$$

Substituting this into equation **(4)** gives:

$$R_0 = \frac{k' R_i}{(1 + C_x / C_2)\, N} \left[\frac{a + bx + C_{1b} + C_{1s}}{C_2} + 1 - \frac{1}{N} - (1 + C_x / C_2)\, N \frac{X_0}{X_{Li}} \right]$$

And differentiating with respect to x:

$$\frac{\partial R_0}{\partial x} = \frac{b\, k' R_i}{(C_2 + C_x)\, N} \qquad [\text{ Ohms }]$$

But this is just b times the derivative $\partial R_0 / \partial C_1$ **(5)**, which we have already had to calculate for the precision part of the evaluation. Hence:

$\partial R_0 / \partial x = b\, \partial R_0 / \partial C_1$

Hence we are in a position to substitute for all the error contributions in equation **(14)** and write our final error function:

$\sigma_Z = \sqrt{\,[\, |\delta \mathbf{Z_0}|_{max}^2 + \sigma_{R0}^2 + 2(\sigma_x\, b\, \partial R_0 / \partial C_1)^2 \,]}$	**15**

From various tests carried out on both versions of the author's bridge; the typical maximum magnitude runout was about ±0.02 Ω, whereas the accuracy with which the reference load resistor can be measured using the Fluke 8060A is ±0.06 Ω. The scale reading and resetting error of the trimmer capacitor makes only a small contribution. This means that the final claimable accuracy is primarily limited by the uncertainty in the reference load resistance. The bridge is a lot better than the resistance meter used, but can only be calibrated by reference to the resistance meter.

5c. Realistic circuit parameters

The bridge evaluation method used here is most accurate when the circuit parameters used for the various calculations are also accurate. Fortunately however, the technique is fairly forgiving, and will not be greatly affected by the differences between nominal and actual component values, provided that reasonable allowances for stray capacitance are made. This circumvents a problem that will confront some constructors, which is that it is not easy to make accurate absolute measurements of capacitances of less than 100 pF. Standard 1 kHz component bridges are far too insensitive, and digital multimeters with capacitance ranges display numbers with a disarming certainty that belies the lack of any relationship with reality. For those who possess the ability to measure inductances of around 10 μH however (e.g., using an SWR analyser such as the MFJ-269 and converting the reactance reading into inductance); there is a way to determine the voltage-sampling network capacitances accurately, and thereby check that all of the other components are within spec. and that everything is working properly.

Equation **(8)** gives the relationship between R_V and the coupled secondary inductance L_i. Here we reproduce it with X_0 set to zero, because that represents the situation when the bridge is balanced.

$$\frac{1}{C_2 R_V} = \frac{(1 + C_x / C_2)\, N R_0}{L_i}$$

This can be rearranged, and at the same time we can observe that the inductance is best determined by the average value of R_V at low frequencies, i.e., R_{Vcal}. Thus:

$L_i = (C_2 + C_x)\, N R_0 R_{Vcal}$	**16**

We can use this expression to calculate L_i from the parameters already available. It should be about 1% less than the measured inductance of the transformer secondary (L_{sec}). Since we have to have a good resistance meter on hand to perform the evaluation, the only dubious parameter here is the quantity $C_2 + C_x$. Hence we can improve the accuracy of $C_2 + C_x$ by tweaking its value in the spreadsheet to make L_i come out about 1% lower than L_{sec}.

For the author's bridge, $L_{sec} = 8.15$ µH. The measured value of the upper voltage sampling capacitor was 4.70 pF. For the Faraday shielded version (i.e., with $C_x = 0$), it was found that setting C_2 in the spreadsheet to 4.9 pF gave L_i as 8.06 µH (i.e., 99% of L_{sec}). Hence, with the layout used, the strays across the upper voltage sampling arm are about 0.2 pF.

The coupled inductance of the current transformer is a strongly conserved parameter. It is not affected by stray capacitance and therefore remains the same when the Faraday shield is removed. Hence, by making measurements on both shielded and unshielded versions of the same bridge, we can determine C_x.

For the unshielded version of the author's bridge, it was found that $C_2 + C_x$ had to be set to 5.35 pF to give $L_i = 8.06$ µH as in the shielded case. This is an increase of 0.45 pF, but not all of it can be attributed to C_x. Removing the shield not only gives rise to stray capacitance from the line to the detector port (C_x), it also gives rise to additional stray capacitance from the line to the summing point (s). We would expect these two additional stray capacitances to be about equal. Hence an increase in $C_2 + C_x$ of 0.45 pF is half due to C_x becoming greater than zero, and half due to an additional contribution to C_2 from strays. Hence our best estimate for C_x for the unshielded bridge is 0.225 pF.

For the shielded bridge ($C_x = 0$), determination of C_2 also allows us to determine C_1. For the unshielded bridge, the ability to determine C_1 also depends on a knowledge of C_x. Hence, in order to make a full characterisation of an unshielded bridge, it is necessary to install a shield at some point and make additional measurements. This, as it will transpire, is the only notable difference between the two versions of the instrument: omitting the shield makes it difficult to estimate a parameter that we don't really need to know.

The resistance balance condition was given earlier as equation **(3)**. Here we reproduce it with X_0 set to zero as before:

$$\frac{C_1}{C_2} + 1 - \frac{1}{N} = \frac{(1 + C_x / C_2)\, N R_0}{k' R_i}$$

This can be rearranged into an expression for C_1. At this point we can also note that if the value of R_0 inserted is the design load resistance R_{0d}, the equation should return an estimate for the final calibration value for C_1. Thus:

$$C_{1cal} = C_2 \left[\frac{(1+ C_x/C_2)\, N\, R_{0d}}{k'\, R_i} - 1 + \frac{1}{N} \right] \qquad (17)$$

In the preceding article[15], the quantity in square brackets was referred to as the transformer constant.

The most diagnostically helpful application of equation **(17)** is to use it to estimate the stray capacitance component of C_1, i.e.:

$$C_{1s} = C_{1cal} - C_{1acal} - C_{1b}$$

C_{1s} needs to be positive and sensible. If it is not, then there is something wrong with the other bridge parameters.

For the Faraday shielded version of the author's bridge, C_{1s} was calculated to be about 5 pF. For the unshielded version, based on a good estimate for C_x, C_{1s} was about 4 pF. These values are not particularly accurate because they are affected by the uncertainties in the other circuit parameters, but they are *reasonable*.

If the analysis shows up an unrealistic value for C_{1s}, it could be simply that some of the circuit components, particularly the fixed capacitors, are out of spec.. If everything appears to be in order however, then there could be a problem with the transformer efficiency factor k'. The point is that k' is affected by resistive loading of the neutralisation winding. The effect is very small provided that $R_n >> R_i / N$. Hence if C_{1s} looks wrong, disconnect one end of the neutralisation trimpot and measure the resistance. R_n should be >20 Ω, and certainly $>> R_i / N = 4.2\ \Omega$.

5d. Estimating the compensating inductance

At very high-frequencies (well above the operating range), the impedance seen at the input (TX) port is essentially the impedance of the upper voltage-sampling arm in parallel with the load resistance. By connecting an MFJ-269 antenna analyser to the TX port; it was found that the unshielded version of the author's bridge exhibited one resonance in the 1.7 MHz to 170 MHz range, this being a series resonance at 145 MHz. Leaving the detector port open-circuit or applying a shorting-plug made no difference, so it must be assumed that the impedance from the summing point (s) to ground is very low at 145 MHz. This means that the resonance is almost entirely governed by L_2 in series with C_2. For the unshielded bridge, the effective C_2 is 5.13 pF, but the part of it in series with L_2 is about 4.7 pF. Hence:

$$L_2 = 1 / [\, (2\pi f_0)^2\, C\,] = 256 \text{ nH}$$

The ratio C_1 / C_2 is given by equation **(17)**. For the unshielded bridge it is 12.14. Hence, the equivalent series inductance of the lower voltage-sampling arm (L_1) is 256 / 12.14 = 21.1 nH. This figure is approximate; but it is realistic and it shows that the task of inductance minimisation has been successfully accomplished.

15 Eval & Opt. Sections 5a and 19 (equation 19.10a)

6. The effect of temperature

On resistance or magnitude error: - negligible

An expression for the bridge load resistance at balance was given as equation (**4**). Since, to a very good approximation, the bridge always balances when $X_0 = 0$, this can be written:

$$R_0 = \frac{k' R_i}{(1 + C_x / C_2)\, N} \left[\frac{C_1}{C_2} + 1 - \frac{1}{N} \right] \qquad (\mathbf{18})$$

None of the parameters in this expression are strongly temperature dependent. The relationship is also governed primarily by the ratio C_1 / C_2. If most of the capacitance C_1 is provided by a capacitor of the same type as C_2 (i.e., silvered mica), then C_1 and C_2 will have roughly the same proportionate temperature coefficient. Hence the capacitance ratio will remain reasonably constant with temperature. It should also be noted that the final magnitude accuracy of the bridge is limited by the accuracy of the resistance meter used to measure the reference load; and that this uncertainty is much larger than the uncertainty due to bridge imperfections.

On phase error:- significant at low frequencies

An expression for the bridge load reactance at balance was given as equation (**10**). At the final calibration setting, this can be written:

$$X_0 = \frac{k' R_i}{2\pi f} \left[\frac{R_{0d}}{L_i} - \frac{1}{N\,(C_2 + C_x)\, R_{Vcal}} \right] \qquad (\mathbf{19})$$

Here, the load reactance depends on L_i, which is directly proportional to the initial permeability (μ_i) of the ferrite material used for the transformer core. A graph of μ_i vs. temperature for type 61 ferrite is reproduced below.

The gradient of the graph in the region of 20°C is +0.32 /K (per Kelvin[16]). If we take the initial permeability as the nominal value of 125, this gives a proportionate temperature coefficient of 0.32 / 125 = 0.0026 /K. Hence, the temperature dependence of L_i is given by the expression:

$$L_{i(T)} = L_{i(Tcal)} \left[1 + 0.0026\, (T - T_{cal}) \right]$$

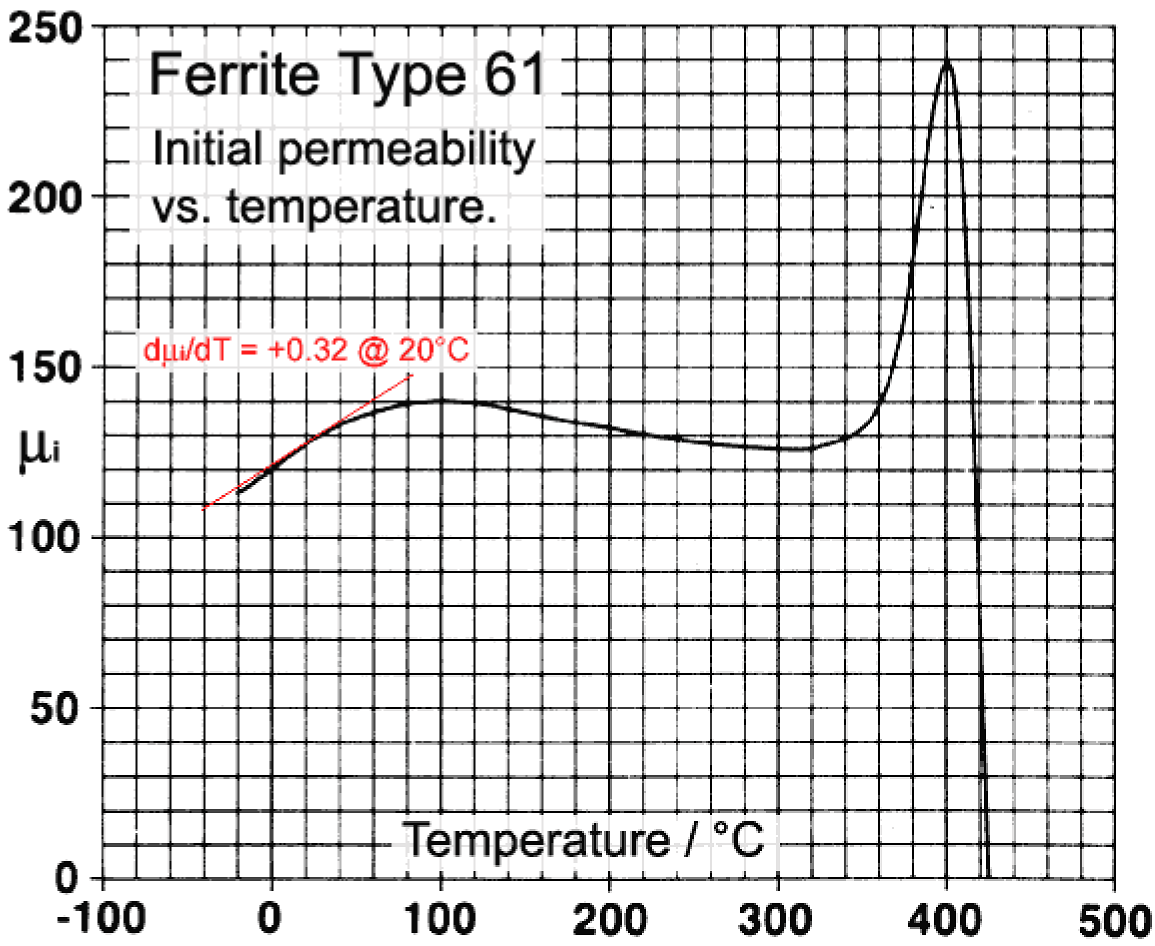
where T is the actual temperature, and T_{cal} is the calibration temperature. We can also represent $L_{i(T)}$ as the inductance at the calibration temperature plus a temperature-dependent error:

$$L_{i(T)} = L_{i(Tcal)} + \delta L_{i(T)}$$

where

$$\delta L_{i(T)} = 0.0026\, (T - T_{cal})\, L_{i(Tcal)}$$

16 For relative temperature measurements, degrees centigrade and Kelvin (capital K) are the same. K is the preferred SI unit, and by using it here we avoid confusion over phase expressed in angular degrees.



The temperature-related load-reactance error is the inductance error multiplied by the rate of change of load-reactance with respect to inductance:

$\delta X_{0(T)} = \delta L_{i(T)}\, \partial X_0 / \partial_{Li}$

i.e.:

$\delta X_{0(T)} = 0.0026\, (T - T_{cal})\, L_{i(Tcal)}\, \partial X_0 / \partial_{Li}$

The derivative is obtained by differentiating equation **(19)**:

$$\frac{\partial X_0}{\partial L_i} = \frac{-\,k'\, R_i\, R_{0d}}{2\pi f\, L_i^2} \qquad \text{[Ohms per Henry]}$$

Thus, evaluating the derivative at the calibration temperature:

$$\delta X_{0(T)} = \frac{-0.0026\, (T - T_{cal})\, L_{i(Tcal)}\, k'\, R_i\, R_{0d}}{2\pi f\, L_{i(Tcal)}^{2}} \qquad \text{[Ohms]}$$

Now recall that $\tan\varphi = X_0 / R_0$. This tangent can be evaluated with R_0 set to R_{0d} without incurring significant error. Also note that, for small angles, the tangent of an angle is linearly proportional to the angle itself. This means that we can evaluate the phase error due to temperature variation, $\delta\varphi_{(T)}$, as an increment to any other source of phase error. Hence:

$$\delta\varphi_{(T)} = \text{Arctan}\left[\frac{-0.0026\,(T - T_{cal})\,k'\,R_i}{2\pi f\, L_{i(Tcal)}} \right]$$

This expression tells us that if the temperature of the transformer core is greater than the calibration temperature there will be a lagging phase error (and vice versa). It also tells us that the magnitude of this error decreases as the frequency increases, and that the temperature sensitivity of the bridge can be reduced by increasing the transformer secondary inductance.

For the author's bridge: $k' = 0.96$, $R_i = 50\ \Omega$, and $L_{i(Tcal)} = 8.06\ \mu H$. This gives:

$$\delta\varphi_{(T)} = \text{Arctan}[\,-2464.3\,(T - T_{cal}) / f\,]$$

By setting $T\text{-}T_{cal} = 1$, we can use this expression to calculate the phase error due to a 1°C (i.e., 1K) temperature increase, and since taking a tangent is a linear operation for small arguments, the result will be a temperature coefficient for the phase error ($d\varphi/dT$) in degrees of angle per Kelvin.

The table below gives an idea of how $d\varphi/dT$ varies with frequency. Also shown are the temperature constraints for a guaranteed phase accuracy of ±0.1° ($\Delta T_{0.1}$) and a guaranteed phase accuracy of ±0.5° ($\Delta T_{0.5}$), these being calculated on the basis that the maximum phase runout error at T_{cal} is ±0.03° (which was the result obtained for the author's bridge).

$\Delta T_{0.1}$ is obtained as follows:

$$|\,d\varphi/dT\,|\,\Delta T + 0.03 = 0.1$$

Hence:

$$\Delta T_{0.1} = 0.07 / |\,d\varphi/dT\,|$$

Similarly:

$$\Delta T_{0.5} = 0.43 / |\,d\varphi/dT\,|$$

f / MHz	dφ / dT / °/ K	$\Delta T_{0.1}$ / K	$\Delta T_{0.5}$ / K
1.6	-0.0882	±0.8	±4.9
2	-0.0706	±1.0	±6.1
3	-0.0471	±1.5	±9.1
5	-0.0282	±2.5	±15
8	-0.0176	±4.0	±24
12	-0.0118	±5.9	±36
17	-0.0083	±8.4	±52
23	-0.0061	±11.5	±71
30	-0.0047	±15	±92

Temperature constraints larger than about ±6 K have been greyed out because these limits are a long way from the temperature at which the temperature coefficient was determined and will therefore not be accurate. Generally, a greyed out temperature constraint can be interpreted as: "no need to worry about it". To put the matter into perspective, the phase error corresponding to an SWR of 1.2:1 is ±7°.

6a. Temperature compensation

Although the bridge described here does not have temperature compensation, it is possible to include such correction by using a thermistor for part of the resistance R_v. As indicated by equation (**16**), R_v is directly proportional to L_i. The proportionate temperature coefficient of L_i is about +2600 ppm / K, and so compensation can be effected by making the temperature coefficient of R_v to be about the same.

For the author's bridge, R_{vcal} was found to be 2748 Ω . A temperature coefficient of 0.0026 /K means that R_v has to increase by about 7 Ω / K to keep the bridge in balance.

Referring to the graph of μ_i vs. T given earlier, note that in the region of 20°C, the gradient decreases as the temperature increases; i.e., the curve is slightly anti-logarithmic in the room-temperature region. Many types of thermistor have a logarithmic characteristic, and are therefore unsuitable for the compensation needed here. Anti-logarithmic thermistors do not appear to be commercially available, and so the best choice is a Linear PTC (positive temperature coefficient) device.

Linear PTC thermistors with a TC of up to +5000 ppM /K are available. The trick will be to find one, with a nominal (25°C) resistance value considerably less than 2.7 kΩ, which increases by about 7 Ω / K. Such a thermistor can then be included in the series combination of resistors that make-up R_v. For best results, the thermistor should be placed in thermal contact with the transformer core (e.g., bonded with a spot of thermally-conductive epoxy resin); and if it is at the top of the resistance chain, one end of it can be connected directly to the summing point (s) end of the secondary winding.

7. Input power-factor

When the bridge is loaded with 50 +**j**0 Ω , the SWR looking into the TX port will be about 1.1 at 30 MHz (i.e., in the worst case). This is due to the combined capacitance of the voltage-sampling and neutralising networks, less the compensation obtained by leaving the through-line unshielded in the gap between the TX socket and the current transformer. The total capacitance burden is about 7 pF. This is also a relatively pure capacitance within the operating frequency range, despite the inductance L_2 in series with the voltage sampling network; the reason being that 4.7 pF has a reactance of -1129 Ω at 30 MHz, whereas L_2, being about 250 nH, has a reactance of only about +47 Ω . Hence, in terms of input power-factor, the effect of L_2 at 30 MHz is to increase the apparent capacitance of C_2 to about 4.9 pF.

In the situation in which the bridge is operated at frequencies well below any internal resonances (which is the intention), the input power-factor can be corrected by inserting a series inductance, between the TX socket and the bridge circuitry, chosen so that:

$$\sqrt{(L/C)} = 50\ \Omega$$

If we take the input capacitance to be 7 pF, the required inductance is 17.5 nH. This can be provided by a tiny coil, which can be made by extending the through line between the TX port and the transformer, stripping off the insulation and winding a few turns around a drill-shank or a screwdriver. The required number of turns can be calculated using the optimised metric version of Wheeler's long-coil formula[17]:

$$L = \frac{\mu_0 \pi r^2 N^2}{\ell [1 + 0.4502 (D / \ell)]} \quad \text{[Henrys]} \quad \pm 0.32\% \text{ when } \ell \geq 0.4\ D$$

e.g., if the 1 mm diameter wire of the through-line is wound around a 4 mm diameter bar, the coil will have an average diameter D of 5 mm. If we make the length of the coil also 5 mm, then $D / \ell = 1$, and:

$$A_L = L / N^2 = \mu_0 \pi r^2 / (1.4502\ \ell) = 3.4 \text{ nH}$$

Hence we need $\sqrt{(17.5 / 3.4)} = 2.3$ turns over a distance of 5 mm.

The connection point for L_2 should be on the transformer-side of any PF-correction coil.

17 Solenoid Inductance Calculation. D W Knight. Section 8c.

8. Test results

Test results for the unshielded and shielded versions of the author's bridge are summarised below. Full details are given in the spreadsheets. The evaluations were conducted in an air-conditioned room at a constant temperature of 22°C.

Statistically, there is no difference between the two versions. Constructors are advised to build the Faraday shielded version for the simple reason that it saves the bother of trying to estimate C_x.

Bridge configuration	Max φ error (const. T)	Max |Z| error (precision)	Max |Z| error (accuracy)	Data analysis spreadsheet
No Faraday shield. R_{Vb} = 2.2 kΩ Carbon film.	±0.032°	±0.039%	±0.131%	**refbrg_D_01.ods**
Faraday shield. R_{Vb} = 2.7 kΩ Carbon film.	±0.030°	±0.041%	±0.133%	**refbrg_D_02.ods**

9. Using the bridge

The bridge can be connected to a diode detector, but this is rather a waste of all the effort that must go into producing it. If a passive detector is to be used, there is not much point in using 3-point phase tracking. Costs can be saved by replacing the neutralising trimpot (R_n) with a fixed resistor of 22 Ω - 27 Ω , and the (slight or non-existent) reduction in accuracy will not be noticed. There are also various other neutralising methods that have a reduced generator power-factor penalty in comparison to quadrature current neutralisation[18].

For the purpose of antenna matching, the best way to use the bridge is in stealth-tuning mode. In that case; the load port is connected to an antenna matching unit (AMU) and thence to an antenna; the TX port is connected to a transceiver; and the detector port is connected to a signal generator or general coverage VFO. Antenna matching takes place with the transceiver in receive mode. The signal generator is tuned to give an audible interfering signal. The AMU is then adjusted to make the signal disappear; at which point the AMU input impedance is 50 +**j**0 Ω and the station is ready to transmit. If the signal generator is mains powered, greatest accuracy will be obtained when a common-mode choke is placed in the line between the generator and the bridge. It is also best practice to terminate the sig. gen. in its preferred load resistance, which can be attached to a T-piece at the detector port.

If the transmitter is keyed into a severe mismatch, a voltage in the 5 V to 10V RMS range can appear at the detector port. It is conceivable, although unlikely, that some signal generators can be damaged in such an event. It is worth noting therefore, that old valve (tube) signal generators are perfectly adequate for stealth tuning; and their internal workings are too brutal to be affected by a few volts applied to the output terminals. Such devices can also be made to appear out of thin air, by moving into the vicinity of a group of radio enthusiasts and uttering the special incantation: "Does anyone have any old valve test equipment they want to get rid of?"

DWK

18 **Eval & Opt**. sections 18c to 18e.