Amplitude response of conventional and maximally-flat current transformers

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Version 1.00, 21st Feb. 2014.

This is an updated version of an HLML article originally published in 2007.
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Abstract
Relative amplitude response measurements over the 1.5 MHz to 30MHz range were performed on a Faraday-shielded radio-frequency current transformer. The transformer was tested in both conventional and maximally-flat circuit configurations and the results agreed with the 'ideal transformer with secondary inductance' model to within the measurement precision of 0.25% (0.02dB). It was found that data could only be made to fit the model when the Faraday shield was maintained at the secondary network reference potential. The effective transformer secondary inductance determined by least-squares fitting to the data was slightly lower than the inductance obtained by direct measurement using a laboratory bridge, the difference being attributable to leakage inductance.

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Experimental method

Instruments that measure voltage at radio frequencies (oscilloscopes, valve / transistor voltmeters, etc.) tend to have the flatness of their frequency responses quoted in dB. Thus, while some might be nearly perfect in the 1.6 MHz to 30 MHz region, there is often no way in which the end-user can find out how flat a particular instrument really is. This presents a difficulty when the task in hand is that of characterising a device that is expected to be flat or, at least, to agree with its theoretical model to within a fraction of a percent. A solution to the problem, which might appeal to those with limited funds, is to use a simple diode detector. Shown right is one of a pair of detectors used in the experiments described below. It consists of a 1N5711 Schottky diode with a 0.1 μF ceramic smoothing capacitor, mounted on the back of a BNC socket. When used to drive a sensitive moving-coil meter via a suitable series resistor, it gives a frequency response that is flat from audio frequencies to several hundreds of MHz (i.e., until the series resonant frequency of the capacitor is reached). To achieve similar performance with an electronic amplifier is difficult, the downside of the simple detector being that large signal levels and correction for the effects of diode non-linearity are required.

Fig. 1
Diode detector

Shown below is a version of the experimental setup used by the author to test the theory of the maximally-flat current transformer. The equipment required is commonly available; but as we will see, with careful procedure and proper data analysis, it will verify the circuit models for both the conventional and the maximally-flat current transformer and, as as an added bonus, it will tell us how best to earth a Faraday shield.

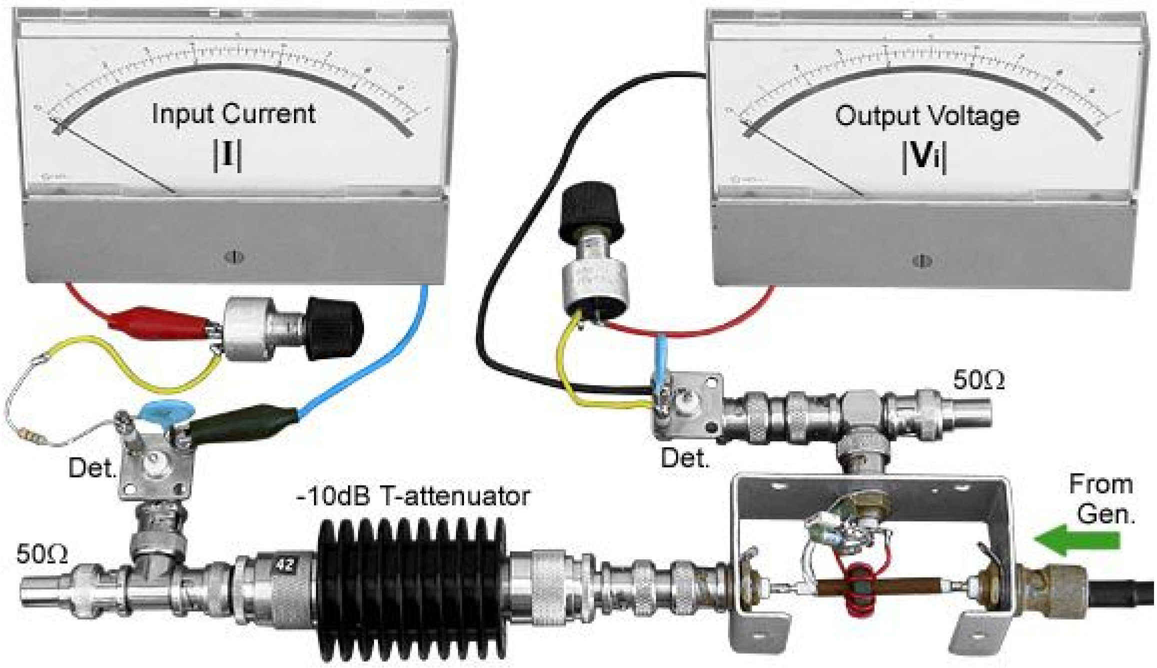

Fig. 2. Test setup for current transformer amplitude-vs.-frequency response.

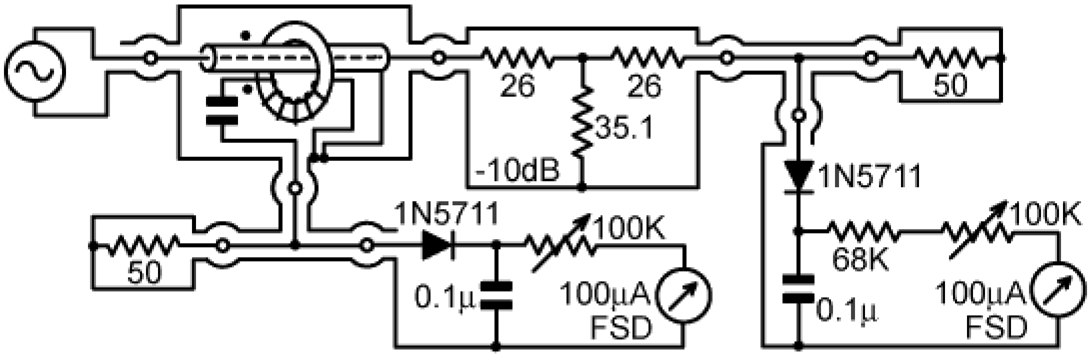

Fig. 3. Test circuit for current transformer amplitude vs frequency response

The basis of the experiment is that, if a precisely constant average power level can be maintained in a load resistor connected to a radio transmitter, then a current transformer in the line to the load will carry a primary current having a precisely constant RMS value. Determining the frequency response of the transformer network is then a matter of measuring its relative output voltage as accurately as possible over a range of frequencies. Using the equipment and circuit shown above, measurements can be made with a transmitter output power of about 10 W, this being obtainable continuously for long periods from a typical 100 W HF radio transceiver. The exact power is not important when making relative measurements, as long as it can be reset accurately at each measurement frequency. Most of the output power (90%) is dissipated in a -10 dB coaxial T-pad attenuator[1], the remainder being delivered to a 50 Ω 1.5 W coaxial terminating resistor. The purpose of the attenuator is to reduce the transmitter output voltage to a level suitable for a diode half-wave detector. A generator delivering 10 W into a 50 Ω load has an output voltage of $\sqrt{(10\times50)} = 22.36$ V RMS. The ratio of the output voltage to the input voltage for the attenuator is given by rearrangement of the expression:

$\eta_{/dB} = -10 = 20\ \mathrm{Log}_{10}(\ V_{out} / V_{in}\ )$

i.e.,

$\mathrm{Log}(\ V_{out} / V_{in}\ ) = -½$

Taking the antilog of both sides (i.e., raising both sides to the power of 10) gives:

$V_{out} / V_{in} = 1/\sqrt{10}$

Since the input voltage is $\sqrt{(10\times50)}$, the RMS voltage at the detector anode is:

$(\sqrt{500}) / (\sqrt{10}) = \sqrt{50} = 7.07$ V

Thus, neglecting diode forward voltage drop, the output after perfect rectification is:

$7.07\ \sqrt{2} = 10$ V.

In practice, a 1N5711 diode will reduce this by about 0.3 V. Consequently, establishing a working power level of around 10 watts is simply a matter of adjusting the meter series resistor (68 kΩ fixed

1 shown in the picture is a narda model 766-10, rated 20 W max, DC - 4 GHz.

+ 100 kΩ variable) so that there is 9.7 V DC across the detector smoothing capacitor when the meter reads full-scale. This can be done by connecting the detector to a variable DC power-supply and using a digital multimeter (DMM) to measure the voltage. Incidentally, unless prior tests are carried out, it is inadvisable to assume that a particular model of DMM can be used to measure DC detector voltages when the transmitter is active. Initial attempts to calibrate the equipment using a digital meter to measure the rectified voltage resulted in erroneous readings (some in error by a factor of 2.2); the problem being that the sampling technique used by some multi-meters is disrupted even by relatively low levels of RF ripple. This undocumented performance limitation of digital meters means that quoted DC-level measurements for RF circuits are not necessarily dependable, whereas the inertial averaging provided by a moving-coil meter is inherently perfect.

Having established the sensitivity of the detector used for input current setting, the next step is to adjust the series resistor for the meter that monitors the transformer output. This is simply a matter of ensuring that the meter gives a high reading but will not go off-scale at any of the measurement frequencies. The nature of the experiment is such that the true parasitic capacitance across the secondary winding (about 4 pF) is not sufficient to affect the output level in the test frequency range (1.5 MHz to 30 MHz); and it was the author's initial (and as it turned out, correct) hypothesis that the so-called 'self capacitance' of the coil is in reality a propagation delay[2], and therefore should not show up in an amplitude-response measurement. Hence both the maximally-flat and the conventional current transformer (should) give maximum output at the highest measurement frequency, and a sensible approach is to set the generator to the highest working frequency and, with the input current meter reading full-scale, set the output level meter to read about 98% of full-scale deflection (FSD). The actual rectified output voltage corresponding to FSD can be determined later using a DC power supply, as can the diode forward voltage drop over the range of currents encountered during the experiment.

Acquiring a data-set involves making a series of measurements at different frequencies, each time setting the input current meter to FSD and reading the output level. Large meters with anti-parallax mirror scales and jewelled (or, better still, taut-suspension) bearings are best for this purpose, but even so there may be difficulties. Using an HF transceiver having a conventional potentiometer for the carrier level control, the author found it practically impossible to set the power level accurately by turning the bare knob. The solution was to improvise a temporary slow-motion drive as shown in **Fig. 4** below. Transceivers that use a digital method to set the carrier level can be expected to be even less cooperative; although if the transceiver uses a separate 13.8 V power supply it should be possible to fine-tune the output by varying the supply voltage between 12 V and 13.8 V. It is, incidentally, by no means guaranteed that a particular transceiver will be good enough for the job. The first transceiver tried by the author (a Kenwood TS-430S) exhibited jitter of several percent in its carrier output and could not be used. The transceiver used for the actual measurements (a TS-930S) showed output fluctuations of around ±1% due to mains-voltage variations, which proved annoying but did not prevent the experiment from being carried out.

2 **Evaluation and Optimisation of current transformer bridges**. D W Knight. http://g3ynh.info/zdocs/bridges/ See **section 16a**.

Fig. 4. Temporary slow-motion drive fitted to a transmitter carrier level control. Precise setting of the power level is all but impossible without such an attachment.

Here a 6:1 friction ball drive is shown coupled to the (outer) carrier control knob of a Kenwood TS930S HF transceiver by means of a short length of PVC tubing (warmed to soften it and then pushed on). An improvised stay for the reduction-drive body is attached to a lower cover retaining screw.

A particular problem when making accurate level readings is meter sticking. This pathology of conventional moving coil meters is evident when different readings are obtained by approaching the set power level from opposite directions. It is not normally noticed, but becomes significant when trying to measure to less than 1/100th of the scale. The author tried several meters before settling on the best ones available. The photograph of **Fig. 2** shows an early version of the experiment. The best data-set was obtained using an old AVO model 8 multimeter set to its 50 μA scale for the input current meter, with the series resistor changed to 150 kΩ fixed + 100 kΩ variable (nominally 194 kΩ). Good data could only be obtained by tapping the input current meter gently (with a pencil or screwdriver handle) while setting the transmitter carrier control, and by tapping the output level meter before taking a reading. The carrier level tended to drift for about 1 minute after the transmitter was switched on, and it was found best to keep it running during the experiment, switching off only briefly to change bands. Keeping the transmitter on also helps to keep the system in thermal equilibrium, although the various coaxial resistors used did not show any change between hot and cold at the 0.1 Ω level, and the detector diodes remained cool to the touch. With the precautions described, it was possible to read the transformer output level to within about ¼ of a division on a 0 - 100 scale, with a repeatability of about ½ of a division. This is consistent with an estimated standard deviation (ESD) of measurement of about 0.0025 .

For each run of the experiment, data were acquired for a maximally-flat current transformer, and then the compensation capacitor (C_h) was removed and the readings repeated. In this way, both a conventional and a maximally-flat current transformer were characterised, and the direct relationship between the parameters of the two models (same secondary inductance, same load resistance) gave rise to an effective doubling of the size of the data-set. This gives greater statistical significance to the data for the purpose of evaluating the experimental method by comparing the observed and the expected variances[3]. From the foregoing discussion of measurement precision, an ESD of fit of about 0.0025 is to be expected if the data agree with the model.

As has already been implied, the object of the exercise here is to make measurements of the relative output voltage of a current transformer, with and without flatness compensation, over a range of frequencies. The measurements are then compared with the relative output voltage predicted by the appropriate model. The relative output is defined as the transfer function magnitude at the measurement frequency divided by the transfer function magnitude at "infinite" frequency (i.e., when the reactance of the transformer secondary inductance is very large and the reactance of the compensation capacitor, when included, is very small). For the maximally-flat current transformer, the relative output is given by the expression[4].

$$\eta_{rel} = 1/\sqrt{[\,(1 + X_{Ch}/X_{Li})^2 + (R_i/X_{Li})^2\,]} \qquad \textbf{1}$$

3 For an explanation of the statistical methods used, see **Scientific Data Analysis**, D W Knight. http://g3ynh.info/zdocs/math/data_analy.pdf

4 See. **The Maximally flat current transformer**. D W Knight. http://g3ynh.info/zdocs/bridges/ Equation (**2**).

(η is Greek "eta") and the condition for maximum flatness is achieved when:

$$C_h = 2\, L_i / R_i^2 \tag{2}$$

When the compensation capacitor C_h is shorted out, its reactance goes to zero, and so the relative output of the conventional current transformer is given by:

$$\eta_{rel} = 1/\sqrt{[\, 1 + (R_i / X_{Li})^2\,]} \tag{3}$$

Before we can compare the measurements with the values given by these expressions however, we must first correct the data for the effects of detector non-linearity, then multiply each observation by a normalisation parameter, which is chosen so that the adjusted measurements are evenly scattered around their predicted values (i.e., the normalisation parameter is used to obtain a least-squares fit). Note that the normalisation parameter is different for the two cases, as can be understood by examining the two circuits below. In the case of the conventional current transformer, the network driving the detector is a DC short circuit; whereas in the case of the maximally flat transformer, the network has a DC resistance R_i. Hence the sensitivity of the maximally flat network is reduced in comparison to the conventional circuit by a factor: $R_D /(R_D + R_i)$. There is also a very small additional reduction in output due to the equivalent series resistance (ESR) of the boost capacitor C_h.

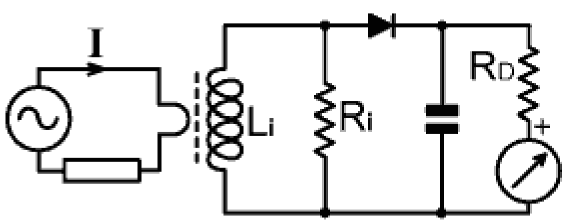

Fig. 5. Conventional current transformer.

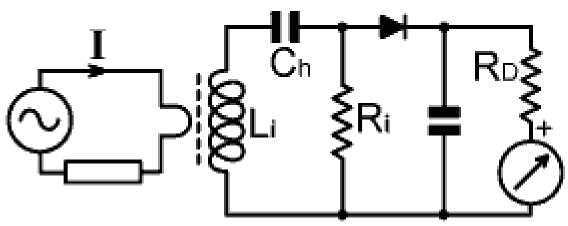

Fig. 6. Maximally flat when $C_h = 2L_i / R_i^2$

Correction for detector non linearity is obtained by connecting the detector to a DC power supply (taking care not to disturb the setting of the meter series resistance) and measuring the diode forward voltage drop (V_f) at a few points over the range of meter readings that were encountered during the experiment. These values are then fitted to an expression of the form:

$$V_f = V_1 \mathrm{Log}(I_f) + V_0$$

and used to correct the actual voltage at the detector output to the voltage that would have been present had the diode been perfect. This means that the detector voltage corresponding to FSD of the meter must also be measured, so that the meter readings can be converted into actual voltages. Correction is then effected by adding the diode forward drop given by the fitting function to the actual measured voltage, then converting back to a relative measurement by dividing by the full-scale voltage. If M_k'' is an actual relative meter measurement (a proportion of FSD, i.e., a value between 0 and 1), and V_{fsd} is the voltage corresponding to a full-scale reading, then the absolute voltage measured is $M_k'' \times V_{fsd}$. Had the diode been perfect, the measured voltage would have been $(M_k'' \times V_{fsd}) + V_f$. Hence, the relative measurement corrected for diode voltage drop is:

$M_k' = (M_k'' V_{fsd} + V_f) / V_{fsd}$

i.e.:

$M_k' = (M_k'' V_{fsd} + V_1 \mathrm{Log}(I_f) + V_0) / V_{fsd}$

but the current flowing in the diode is $M_k''{\times}I_{fsd}$, where I_{fsd} is the FSD meter current. Hence:

$M_k' = (M_k'' V_{fsd} + V_1 \mathrm{Log}(M_k'' I_{fsd}) + V_0) / V_{fsd}$

Note incidentally, that it does not matter if I_{fsd} is only the nominal (uncalibrated) value rather than the true FSD meter current, provided that the I_f values used to produce the diode model were obtained from the same meter. Note also that the static average diode forward voltage drop, obtained by making measurements with a DC power supply, might not correspond exactly to the dynamic average forward drop that occurs when the detector has an AC input. In other measurements made by the author however[5], it was possible to infer that this effect is negligible.

Finally, the corrected observations are all multiplied by a common fitting parameter (p say) that scales the dataset in order to obtain the best fit to the model. Thus, a corrected observation is given by:

$M_k = p\, M_k'$

i.e.,

$M_k = p\, (M_k'' V_{fsd} + V_1 \mathrm{Log}(M_k'' I_{fsd}) + V_0) / V_{fsd}$	**4**

where M_k'' is a raw observation. As mentioned earlier, p for the maximally flat transformer will be larger than for the conventional transformer by a factor of approximately: $(R_D + R_i)/ R_D$; R_D being the total meter series resistance. p is adjusted so that:

$$\sum_{k=1}^{n} (M_k - \eta_k) = 0$$

η_k being the corresponding theoretical value obtained from equation (**1**) or (**3**) as appropriate.

Diode correction

Several experimental runs were performed, because various problems had to be ironed out. These all involved slightly different working power levels and various different microammeters, and consequently required slightly different diode corrections. For the purpose of illustration it will be sufficient to describe how only one of these correction functions was obtained. For the experiment in question, the current transformer output detector was connected to a 100 µA meter via a 100 kΩ variable resistor that had been adjusted to give 98 µA (nominal, read from the meter scale) at the working power level with the system operating at 30 MHz. On removing the detector from the circuit and connecting it to a 0 - 30 V bench power supply, it was found that the voltage across the

5 **Current transformer efficiency factor**. D W Knight. http://g3ynh.info/zdocs/bridges/

smoothing capacitor was 2.57 ±0.02 V (measured using a DVM) with the meter reading full-scale. The range of currents that had been encountered during the preceding experiment was 77.5 μA to 100 μA. The corresponding diode forward voltage drops at four points enclosing this range were (±0.002 V):

I_f / μA	100	90	80	70
V_f / V	0.293	0.291	0.287	0.284

The forward currents being as read from the meter scale. These values were fitted to a regression line[6], giving the diode function as:

$V_f = 0.026117601 \, \mathrm{Log_e}(I_f / \mu A) + 0.172947811$ [Volts]

Details of the fitting process are shown below and the formulae used can be inspected in sheet 2 of the spreadsheet **maxflat_test1.ods**.

	A	B	C	D	E	F	G	H	I	J
1	Diode correction function						Vf = V0 + V1 Ln(If/uA)			
2	If	Vf	Ln(If)	xi yi	xi^2			Vf calc	obs-calc	ei^2
3	/ uA	(=yi)	(=xi)						(=ei)	
4	**100.0**	**0.2930**	4.605	1.349	21.208	V0=a	**0.172947811**	**0.293224**	-0.000224	0.000000050
5	**90.0**	**0.2910**	4.500	1.309	20.248	esd a	0.008287976	**0.290472**	0.000528	0.000000279
6	**80.0**	**0.2870**	4.382	1.258	19.202	V1=b	**0.026117601**	**0.287396**	-0.000396	0.000000157
7	**70.0**	**0.2840**	4.248	1.207	18.050	esd b	0.001868399	**0.283908**	0.000092	0.000000008
8	sum >	1.1550	17.736	5.123	78.708	D	0.282973317		0.000000	0.000000494
9									esd fit >	**0.000496950**

Hence, for this particular experimental run (putting the numbers into equation (4), a corrected meter reading is given by the expression:

$M_k = p \, (2.57 \, M_k'' + 0.026117601 \, \mathrm{Log_e}(100 \, M_k'') + 0.172947811) / 2.57$

Since any errors in the diode function are small in comparison to the measurement precision, and p is always close to 1, the estimated standard deviation (ESD) of a corrected measurement remains approximately the same as that of a raw measurement, i.e., about 0.0025.

Data collection

A number of experiments were carried out using a 1:12 Faraday-shielded current transformer wound on an Amidon FT50-61 bead. The diameter of the enamelled copper secondary winding wire was 0.9 mm, and the measured inductance at 1.5915 MHz was 8.45 ±0.21 μH. The current transformer load resistor R_i was 49.8 ±0.2 Ω in all cases. The detector input resistance, of course, slightly reduces the effective value of R_i. but since the effect is small in these experiments, and identical for the conventional and maximally flat circuit configurations during an experimental run, it was neglected (i.e., allowed to become part of the fitting parameter p).

Only two of the experiments will be reported, the others being concerned with resolving

6 See **Scientific Data Analysis** (already cited).

practical problems such as meter sticking and variation in the transmitter output level.

The first valid experiment, with the current transformer mounted as shown in **Fig. 9** below, was a failure in the sense that the data did not fit the models; but it provided important information nevertheless. The graphs of relative output vs. frequency and deviation from theory are shown below. The data and details of the data adjustment procedure are given in the spreadsheet **maxflat_test1.ods** .

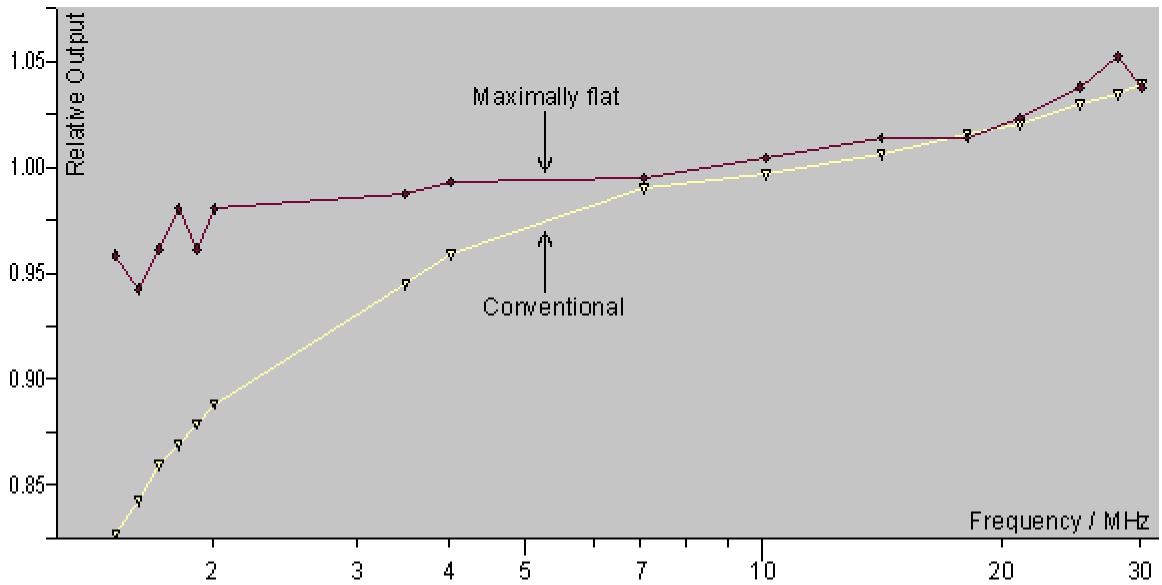

Fig. 7: Amplitude vs frequency response of the current transformer setup shown in **Fig. 9**.

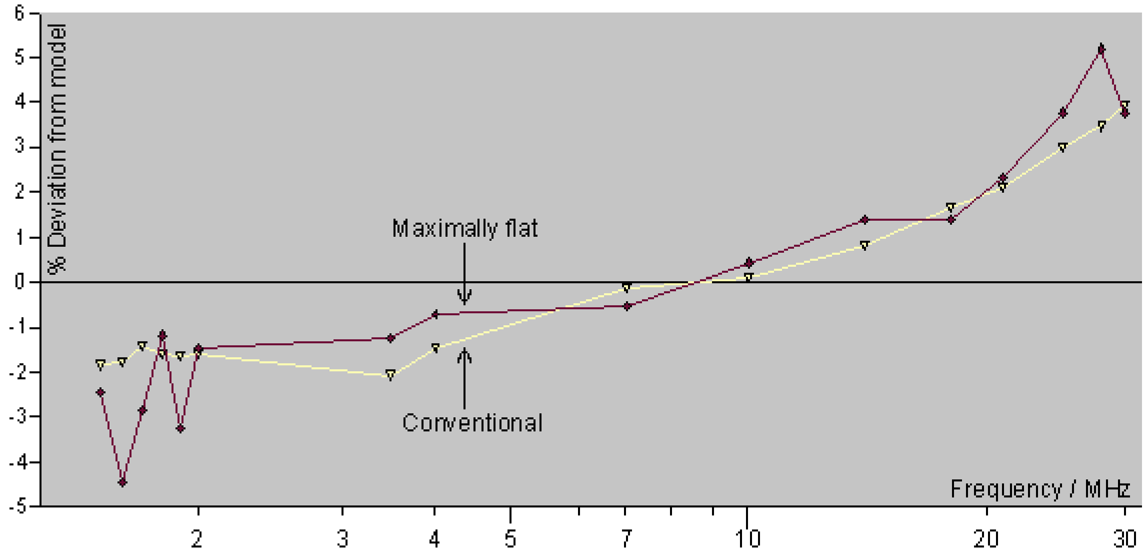

Fig. 8: Deviation from theory for the current transformer setup shown in **Fig. 9**.

The problem with the first experiment is that the data show a steadily rising frequency response. The extent of the deviation from theory is apparent in the graph of **Fig. 8** above; which shows a rise of about 7% over the 1.5 MHz to 30 MHz range, with no statistically significant difference in this respect between the conventional and maximally flat configurations. This is obviously an artifact, a supposition confirmed by measuring the actual voltages involved against the input current[7]. By making absolute (rather than relative) measurements, it was found that the apparent transfer efficiency of the transformer exceeded 100% at 30 MHz, which is of course impossible. The cause of the problem can be deduced by considering the earthing arrangement in **Fig. 9** below.

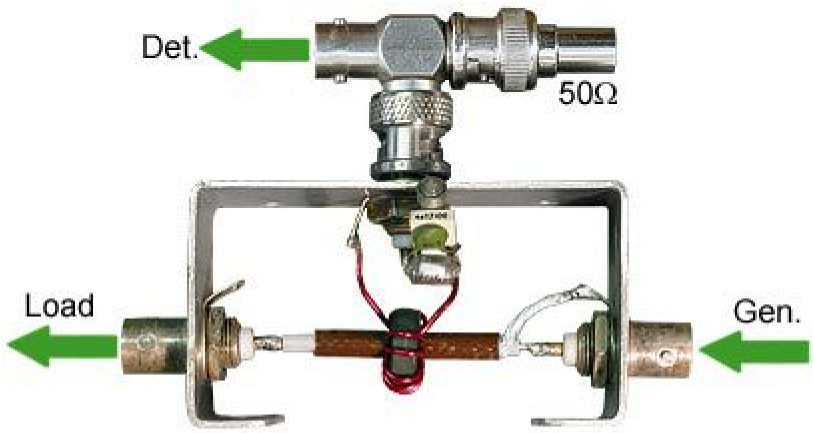

Fig. 9:
Faraday shield earthing arrangement giving a current transformer output response that rises with frequency relative to the theoretical model.

The experimental defect lies in the fact that the primary current flows in the aluminium bracket on which the transformer is mounted and the Faraday shield is grounded at the generator port. The inductance of the bracket is small, but nevertheless finite; and so a potential difference, that rises with frequency, exists between the ports. Hence the Faraday shield potential differs from the detector reference potential (i.e., the ground connection at the detector socket), and some of this difference is coupled to the detector by the capacitance between the Faraday shield and the transformer secondary winding. The reactance of this parasitic capacitance, of course, falls with frequency, and so the coupling also increases with frequency. The problem was cured by rewiring the jig as shown in **Fig. 10** below. Fortunately this issue was resolved at an early stage in the author's experiments because the corollary is that current transformers do not behave according to theory unless the Faraday shield is maintained at the secondary network reference potential.

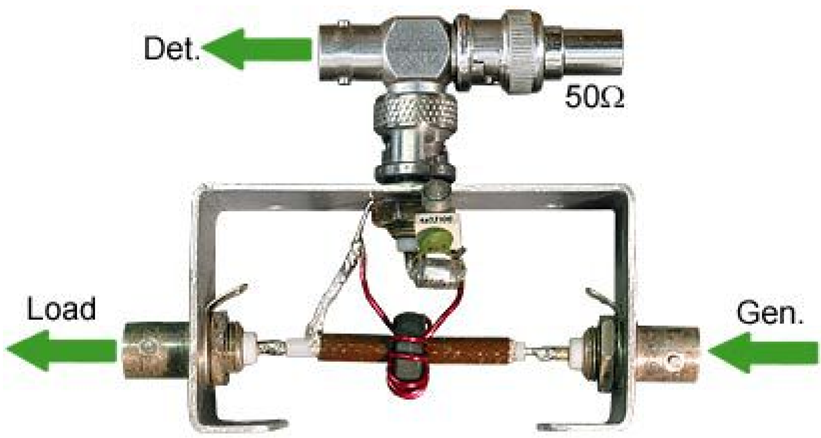

Fig. 10:
Faraday shield earthing arrangement giving an amplitude vs frequency response that is in agreement with the theoretical model.

7 Using the method given in '**Current transformer efficiency factor**' (already cited)

Final normalised data for the Faraday-shield earthing configuration shown above are plotted in **Fig. 11** below. It is obvious that the flatness compensation scheme has worked, and so a detailed analysis follows.

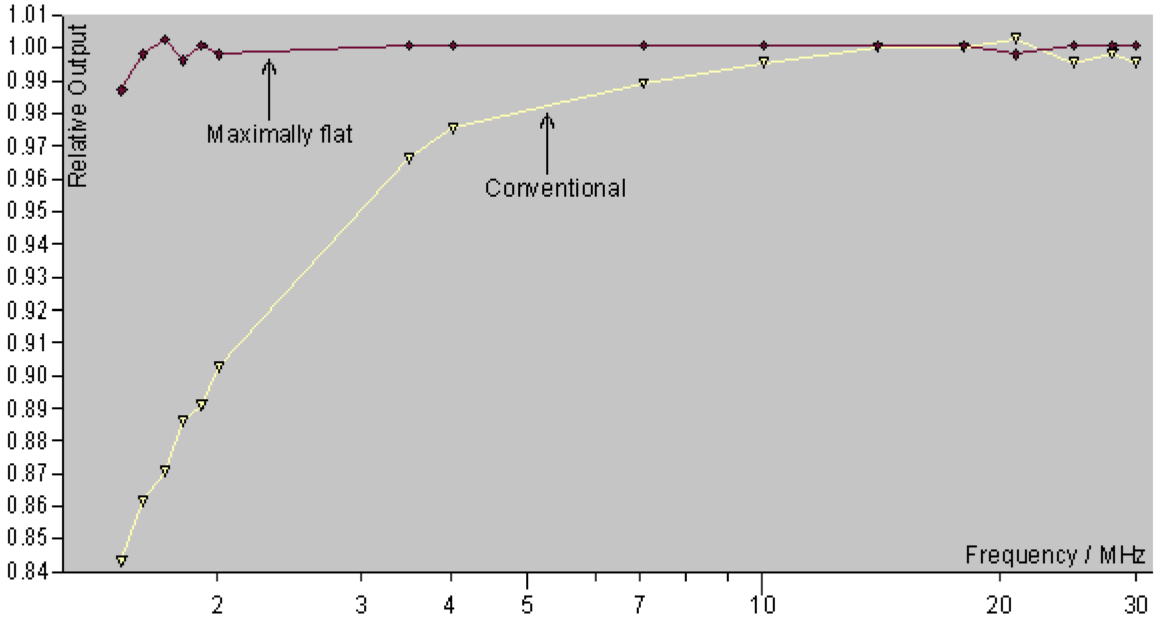

Fig. 11: Amplitude vs frequency response of the current transformer setup shown in **Fig. 10**. Labeling the two curves to distinguish the maximally-flat from the conventional is perhaps not necessary.

The data were first corrected for diode forward voltage drop, and the scaling parameter p was adjusted for minimum standard deviation of fit (reduced χ^2). It was then found that the fit to the data could be improved by varying L_i and C_h slightly. The compensarion capacitor was removed from circuit and its capacitance was measured to be 6.25 ±0.16 nH at 1.5915 MHz, in agreement with the experiment but about 7% low of its value read earlier from a digital capacitance meter. The optimal secondary inductance value was found to be around 8.3 µH, as opposed to the measured inductance of 8.45 ±0.21 µH at 1.5915 MHz. The adjusted inductance value is, of course, still in agreement with the measured value, but the laboratory bridge used for the measurement[8] was known to be more accurate than its specification and the discrepancy was considered to be genuine. The cause of the problem remained unknown until a technique was developed for extracting the effective secondary inductance and 'self-capacitance' of a current transformer from bridge-parameter measurements[9]. It then became apparent that the effective inductance is always less than the measured inductance, the difference being a few %. Steps were taken to eliminate all causes of systematic error, but the difference persisted; leading to the conclusion that it was due to leakage inductance; i.e., the effective inductance is the coupled inductance, whereas the measured inductance is the sum of the coupled inductance and the secondary leakage inductance.

8 Hatfield LE300/A

9 Evaluation and optimisation of current transformer bridges (already cited).

Extracting the coupled secondary inductance

On the basis that that it is legitimate to adjust the fit for leakage inductance, the data were re-analysed in order to extract the effective or 'coupled inductance'. Working with the data for the conventional current transformer, this was done by noting that a normalised amplitude measurement (M) should be in agreement with equation **(3)**, i.e.,

$$M = 1/\sqrt{[\,1 + (R_i/X_{Li})^2\,]}$$

but

$$M = p\,M'$$

i.e., a normalised measurement is a measurement corrected for diode forward drop (M') multiplied by a scaling parameter. Hence:

$$M'\,p = 1/\sqrt{[\,1 + (R_i/X_{Li})^2\,]}$$

This rearranges to:

$$\frac{1}{(M')^2} = p^2 + \frac{p^2 R_i^2}{(2\pi f\,L_i)^2}$$

which is in the form $y = a + bx$, with $y = (1/M')^2$, $a = p^2$, $x = 1/(2\pi f)^2$ and $b = (p R_i / L_i)^2$. Using these identities, the data were subjected to a weighted linear regression analysis[10]. For the purpose of such an analysis, the uncertainty of a given y value (y_k) is derived as:

$$\sigma_{yk} = |\partial y/\partial M'|\,\sigma_{M'}$$

where $\partial y/\partial M' = -2 / (M')^3$

Hence:

$$\sigma_{yk} = 2\,\sigma_{M'} / (M')^3$$

and the fitting weight for an observation y_k is $1/\sigma_{yk}^2$. How the fitting was done can be determined by examining sheet 2 of the spreadsheet **Maxflat_test2.ods**. The inductance is obtained from the fitting parameters, i.e.:

$$b = (p R_i / L_i)^2 \quad \text{and} \quad p = \sqrt{a}.$$

which give:

$$L_i = R_i \sqrt{(a / b)} \qquad \ldots\ldots \textbf{(5)}$$

The variance (i.e., the square of the uncertainty) in L_i, on the assumption of minimal correlation between a and b, is given by:

$$\sigma_L^2 = (\sigma_{Ri}\,\partial L/\partial R_i)^2 + (\sigma_a\,\partial L/\partial a)^2 + (\sigma_b\,\partial L/\partial b)^2$$

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10 See: **Scientific data analysis** (already cited).

where, by differentiating equation (5):

$\partial L/\partial R_i = \sqrt{(a / b)}$

$\partial L/\partial_a = (R_i/\sqrt{b}) / (2\sqrt{a}) = R_i / [\ 2\sqrt{(ab)}\ ]$

$\partial L/\partial b = -(R_i \sqrt{a})/(\ 2\ b^{3/2})$

The uncertainty in R_i (σ_{Ri}) was 0.2 Ω . The uncertainties in a and b were obtained from the fit. The inductance measurement thereby obtained was $L_i = 8.328 \pm 0.034$ µH, the uncertainty in this value being due largely to the uncertainty in R_i. The secondary leakage inductance is of course the difference between this and the directly measured inductance (8.45 ±0.21 µH), but since the uncertainty in the difference is $\sqrt{(0.034^2 + 0.21^2)}$, we get the result

$L_L = 122 \pm 213$ nH

i.e., the uncertainty is bigger than the measurement, and so the data are not accurate enough to determine it.

Having determined L_i from the fit, the value obtained was fed back into the comparison between the data and the model (see sheet 1 of **Maxflat_test2.ods**). The numer of degrees of freedom in the data was thereby reduced by 1, and the calculation of the ESD of the fit adjusted accordingly. The maximally flat transformer data gave $\chi^2/15 = 0.95$, and the conventional transformer data gave $\chi^2/14 = 0.98$, consistent with a reasonable initial estimate for the uncertainty of an observation (0.0025). The pattern of residuals (expressed as percentage deviation) is shown in **Fig. 12** below. Observe that the Y-axis covers an interval of 1%. The fluctuations are typical of rounding error, there being only 4 possible recorded values for a measurement in this interval (ending in 0, 2 or 3, 5, 7 or 8), the decision between 2 or 3 and 7 or 8 being made by tossing a coin.

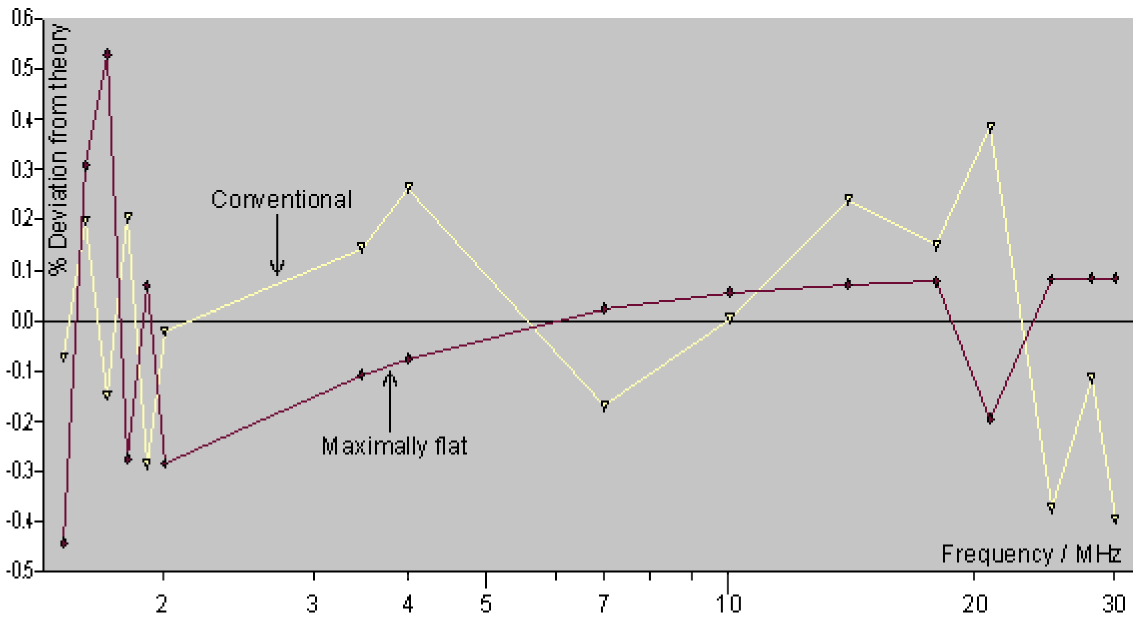

Fig. 12: Data scatter for measurements on the current transformer setup shown in **Fig. 10**.

Note that the reduced χ^2 for both data sets is close to 1, which means that the scatter is attributable to experimental error NOT variation in the amplitude response.

For a secondary inductance of 8.328 μH and a load resistance R_i of 49.8 Ω, the optimum value for the boost capacitance, given by $C_h = 2\ L_i / R_i^2$ is 6.72 nH. The actual value installed was slightly lower than this, at 6.25 nH, resulting in a slight over-boost in the theoretical value for η_{rel} (see sheet 1 of **Maxflat_test2.ods**). This over-boost amounted to 0.2%, indicating that the selection of C_h for a maximally-flat current transformer is not critical within about 10%.

Summary

● The maximally-flat current transformer is a viable solution to the problem of low-frequency amplitude error in RF ammeters.

● Selection on test for the boost capacitor is not required.

● When the 'true' secondary network parallel capacitance is small, the output amplitude vs. frequency response (but not the phase response) of a current transformer can be computed on the basis that the transformer secondary reactance is purely inductive.

● The effective secondary inductance of a current transformer is smaller that the inductance obtained by direct measurement of the coil. The difference is due to leakage inductance. Allocation of different symbols to these to these two quantities is recommended, e.g.:

L_{sec} = measured inductance of the secondary winding.

L_i = effective secondary parallel inductance or 'coupled inductance'.

L_{sec} serves as an approximation for L_i in initial design calculations, but allowance or adjustment for the difference might be required..

● The Faraday shield of a current transformer must be grounded at the secondary network reference point in order to obtain conformance to the 'ideal transformer with secondary inductance' frequency-response model.

DWK, 2005, 2007, 2014