Analysis of Herzog's LF compensation method for current-transformers[1]

By David Knight

Abstract

A Low-frequency phase compensation scheme for RF current-transformers described in **US Patent No. 4739515** (1988) is analysed. Part of the transformer secondary load resistance is placed in series with a capacitor, and this serves to 'unload' the transformer at low frequencies. The method gives an improvement in phase performance but degrades amplitude performance, leading to unnecessary low-frequency balance-point error in reflectometer applications.

Herzog's LF compensation method

In the design of current-transformer bridges for RF applications, it is conventional to correct for the falling reactance of the current-transformer secondary winding at low frequencies by modifying the voltage sampling network. It is feasible however, that compensation might be obtained by modifying the current-transformer secondary loading network; particularly by shunting the transformer output with two resistors in parallel and placing a capacitor in series with one of those resistors. As the reactance of the capacitor increases with diminishing frequency, the magnitude of the load impedance rises, thereby causing the output voltage to increase. This 'unloading' method was patented by Will Herzog in 1988 [**US Pat. No. 4739515**], and further background information is given in a contemporary article[2]. How to choose the component values for the secondary network is not a straightforward matter however, and neither of Herzog's documents offer a design procedure. Here however it will be shown that the best compensation is achieved when the circuit exhibits a condition known as *critically-damped resonance*.

For the purpose of low-frequency analysis, we will neglect the propagation delay and secondary parasitic capacitance of the current transformer. Thus, in the circuit shown in **fig. 1**, the current analog $\mathbf{V_i}$ is given by:

$$\mathbf{V_i} = (\mathbf{I}/N_i)[R_i \,//\, \mathbf{j}X_{Li} \,//\, (R_h + \mathbf{j}X_{Ch})]$$

and when the generator is loaded with a resistance R_0 :

$$\mathbf{I} = \mathbf{V} / R_0$$

and so:

$$\mathbf{V_i} = (\mathbf{V} / N_i R_0)[R_i \,//\, \mathbf{j}X_{Li} \,//\, (R_h + \mathbf{j}X_{Ch})]$$

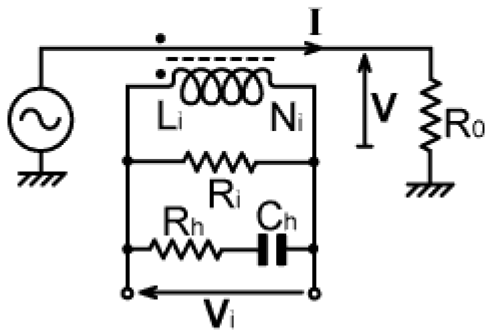

Fig. 1

If we define the transfer function as:

$$\boldsymbol{\eta}_i = \mathbf{V_i} / \mathbf{V}$$

then:

1 Version 1.00, 19th Feb. 2014. Derived from an HTML article first published in 2007. Please check the author's website to make sure that you have the most recent version of this document and its accompanying spreadsheet file: http://g3ynh.info/zdocs/bridges .

2 **VSWR Bridges**, Will Herzog (K2LB), Ham Radio, March 1986, p37-40.

$\boldsymbol{\eta}_i = [\ R_i // \mathbf{j}X_{Li} // (R_h + \mathbf{j}X_{Ch})\ ] / N_i R_0$

The network impedance inside the square brackets of this expression is cumbersome when expanded, and so we will work with the reciprocal transfer function and treat it as an admittance, i.e.:

$1/\boldsymbol{\eta}_i = N_i R_0 [\ (1/R_i) + (1/\mathbf{j}X_{Li}) + 1/(R_h + \mathbf{j}X_{Ch})\ ]$

Multiplying numerator and denominator of the rightmost term by the complex conjugate of its denominator then gives:

$1/\boldsymbol{\eta}_i = N_i R_0 [\ (1/R_i) + (1/\mathbf{j}X_{Li}) + (R_h - \mathbf{j}X_{Ch}) / (R_h^2 + X_{Ch}^2)\ ]$

which, remembering that $1/\mathbf{j} = -\mathbf{j}$, can be separated into real and imaginary parts:

$1/\boldsymbol{\eta}_i = N_i R_0 \{\ (1/R_i) + [\ R_h/(R_h^2 + X_{Ch}^2)\ ]\ \mathbf{-j}[\ (1/X_{Li}) + X_{Ch}/(R_h^2+X_{Ch}^2)\ ]\ \}$	**1**

Now observe that at infinite frequency, $X_{Li} \rightarrow \infty$ and $X_{Ch} \rightarrow 0$, hence the imaginary part of the function vanishes and the real part reduces to:

$1/\eta_\infty = N_i R_0 \{\ (1/R_i) + (1/R_h)\ \}$

from which it can be seen that at infinite frequency, the current transformer load is simply $R_i // R_h$. Thus, when designing a bridge, it is $R_i // R_h$, rather than R_i on its own that must be used when determining the ratio of the voltage sampling network impedances. Hence, if a voltage sampling network is to be pre-chosen, we might, for example, decide to impose the condition $R_i // R_h = 50\ \Omega$. This gives us a link to standard design procedure, but before we can determine the actual resistor values we must explore the conditions under which circuit resonance can occur.

The current transformer secondary inductance L_i will resonate with the compensation-network capacitance C_h if the imaginary part of equation (**1**) can go to zero at some finite frequency, i.e., when:

$(1/X_{Li}) + X_{Ch} / (R_h^2 + X_{Ch}^2) = 0$

This expression can be expanded and rearranged thus:

$[\ 1/(2\pi f\, L_i)\ ] - 1/[2\pi f\, C_h (R_h^2 + X_{Ch}^2)\ ] = 0$

$1/L_i = 1/[\ C_h (R_h^2 + X_{Ch}^2)\ ]$

$R_h^2 + X_{Ch}^2 = L_i / C_h$

$X_{Ch}^2 = (L_i/C_h) - R_h^2$

$X_{Ch} = \pm\sqrt{[(L_i/C_h) - R_h^2\ ]}$

Since capacitive reactance is negative, the negative sign of the square-root is appropriate, hence:

$1/(2\pi f_0 C_h) = \sqrt{[(L_i/C_h) - R_h^2]}$

$$f_0 = 1/\{ 2\pi\ C_h \sqrt{[(L_i/C_h) - R_h^2]} \}$$

Multiplying C_h into the square-root bracket puts the expression into a form similar to the standard resonance formula:

$$f_0 = 1/\{ 2\pi \sqrt{[L_i C_h - (R_h C_h)^2]} \}$$

This is a resonance condition that can have imaginary solutions. Imaginary resonance[3] is a situation in which the phase angle of the impedance of a network can never cross the zero axis. In this case, imaginary resonance occurs when $R_h^2 > L_i/C_h$, and real resonance can occur when $L_i/C_h > R_h^2$. What we want however is the critically damped condition, between these two operating regions, this being the choice that will keep the phase angle close to zero over the widest possible frequency range. Thus critical damping occurs when:

$$L_i / C_h = R_h^2 \qquad \textbf{2}$$

Hence, for a given current transformer with a secondary inductance L_i, the choice of R_h determines C_h. This still leaves us with the problem of how to choose R_h, but we can settle that matter by performing some calculations. We should be suspicious however; of the fact that the parallel resistance R_i does not appear in equation (**2**), and that its value can therefore be set arbitrarily. This suggests that R_i is redundant.

In order to evaluate Herzog's LF compensation scheme, we can imagine that the transformer is used in a bridge that, if it were to operate ideally, would have a current transfer function at balance:

$\eta_\infty = (R_i // R_h) / N_i R_0$

Since it does not operate ideally, it has an actual transfer function:

$\eta_0 = [R_i // \mathbf{j}X_{Li} // (R_h + \mathbf{j}X_{Ch})] / N_i \mathbf{Z}_{bal}$

where $\mathbf{Z}_{bal}$ is the actual load required in place of R_0 in order to balance the bridge. Note that this transfer function is real, and numerically identical to the ideal case transfer function, because the process of adjusting $\mathbf{Z}_{bal}$ to balance the bridge will give $\mathbf{Z}_{bal}$ the same phase angle as the current transformer secondary network impedance $[R_i // \mathbf{j}X_{Li} // (R_h + \mathbf{j}X_{Ch})]$. The ratio of two impedances having the same phase angle is scalar[4]. Hence, equating the ideal and actual transfer functions:

$(R_i // R_h) / N_i R_0 = [R_i // \mathbf{j}X_{Li} // (R_h + \mathbf{j}X_{Ch})] / N_i \mathbf{Z}_{bal}$

which gives:

$$\mathbf{Z}_{bal} = [R_0 /(R_i // R_h)] [R_i // \mathbf{j}X_{Li} // (R_h + \mathbf{j}X_{Ch})]$$

3 **AC Theory**, D W Knight. www.g3ynh.info/zdocs/AC_theory/ See **section 21**.
4 **AC Theory** (cited above), **section 24.8**.

Notice that if we choose $R_0 = 50\ \Omega$ and $R_i // R_h = 50\ \Omega$, then the factor $R_0 / (R_i // R_h)$ is unity. Notice also that we have already expanded the network impedance in the second square bracket during the process of obtaining equation (1), and so we can re-write the the expression above:

$$\mathbf{Z}_{bal} = [\, R_0/(R_i // R_h)\,] / \{ (1/ R_i) + [\, R_h/(R_h^2 + X_{Ch}^2)\,] - \mathbf{j}[\, (1/X_{Li}) + X_{Ch}/(R_h^2 + X_{Ch}^2)\,] \}$$

The way in which $\mathbf{Z}_{bal}$ changes with frequency characterises the bridge, because the ratio $|\mathbf{Z}_{bal}| / R_0$ gives the magnitude error in the balance condition, and the phase of $\mathbf{Z}_{bal}$ gives the phase error. Hence, what we need to do now is to obtain expressions for the magnitude and phase of $\mathbf{Z}_{bal}$ and calculate these quantities for various values of R_i and R_h. To that end we can write:

$$\mathbf{Z}_{bal} = k / (G + \mathbf{j}B)$$

where:

$$k = R_0/(R_i // R_h)$$

$$G = (1/R_i) + R_h/(R_h^2 + X_{Ch}^2)$$

and

$$-B = (1/X_{Li}) + X_{Ch}/(R_h^2 + X_{Ch}^2)$$

Hence:

$$\mathbf{Z}_{bal} = k\, (G - \mathbf{j}B) / (G^2 + B^2)$$

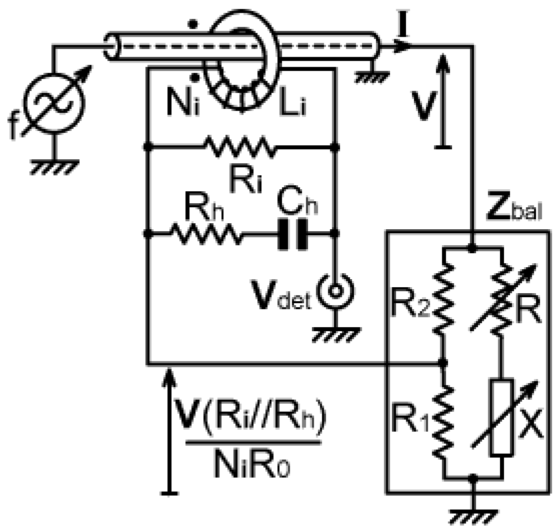

Fig. 2

which gives:

$$|\mathbf{Z}_{bal}| = k\, [\, \sqrt{(G^2 + B^2)}\,] / (G^2 + B^2)$$

i.e.,

$$\boxed{|\mathbf{Z}_{bal}| = k / \sqrt{(G^2 + B^2)}}$$

and the phase angle is:

$$\boxed{\varphi = \mathrm{Arctan}(-B/G)}$$

Now, setting up a spreadsheet (**Herzog_LF.ods**) with columns for Log(f), f, G, -B, $|\mathbf{Z}_{bal}|$ and φ, we can create graphs and tables that show how $|\mathbf{Z}_{bal}|$ and φ change with frequency. Shown below are the results of calculations performed for a current transformer with $L_i = 10\ \mu H$, and a secondary load at high frequencies ($R_i // R_h$) of $50\ \Omega$. The characteristic resistance R_0 is also $50\ \Omega$, making $k = R_0 / (R_i // R_h) = 1$. The capacitance C_h is calculated using the critical damping condition (equation 2), and the required values for various combinations of R_i and R_h are given in **table 1**. Also given are the required capacitances when $L_i = 20\ \mu H$, showing that they scale in proportion to the inductance. It transpires that if the transformer secondary inductance is multiplied by some factor, then the frequency at which a given phase or magnitude error occurs is divided by the same factor. Consequently, the graphs for the 10 µH case can be made to serve for the 20 µH case (say) by drawing a new frequency scale with all of the numbers divided by 2.

Table 1. Compensation capacitance for various choices of R_i and R_h

R_i / Ω	R_h / Ω	$C_h = L_i / R_h^2$ (L_i=10 µH)	$C_h = L_i / R_h^2$ (L_i=20 µH)	
50	∞	0	0	Uncompensated
100	100	1000 pF	2000 pF	
150	75	1778 pF	3555 pF	
300	60	2778 pF	5556 pF	
1300	52	3698 pF	7396 pF	
∞	50	4000 pF	8000 pF	Maximum LF boost.

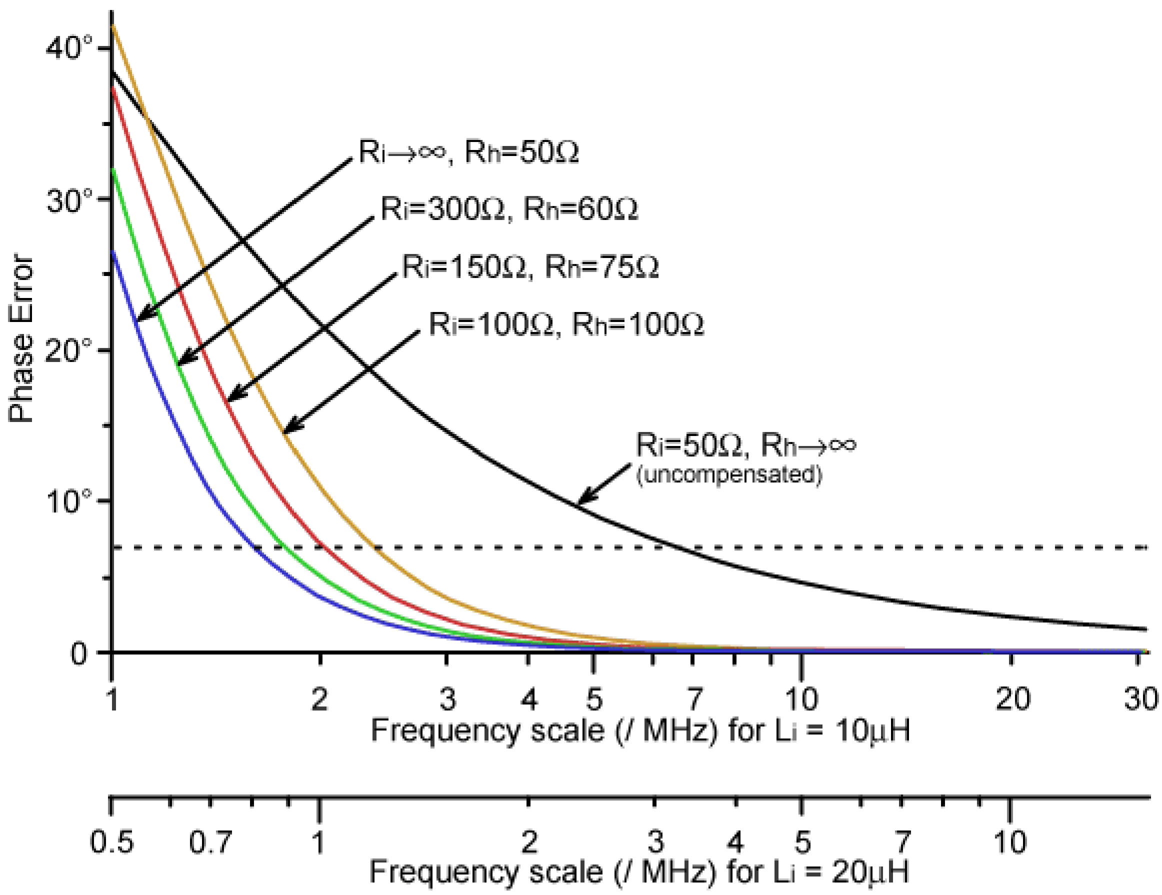

Fig 3. Phase error of bridges with Herzog compensation and critical damping of the current transformer secondary network resonance. Upper frequency scale is for L_i = 10 µH , lower frequency scale is for L_i = 20 µH. The dotted line at 7° represents an arbitrarily chosen limit for 'acceptable' phase performance.

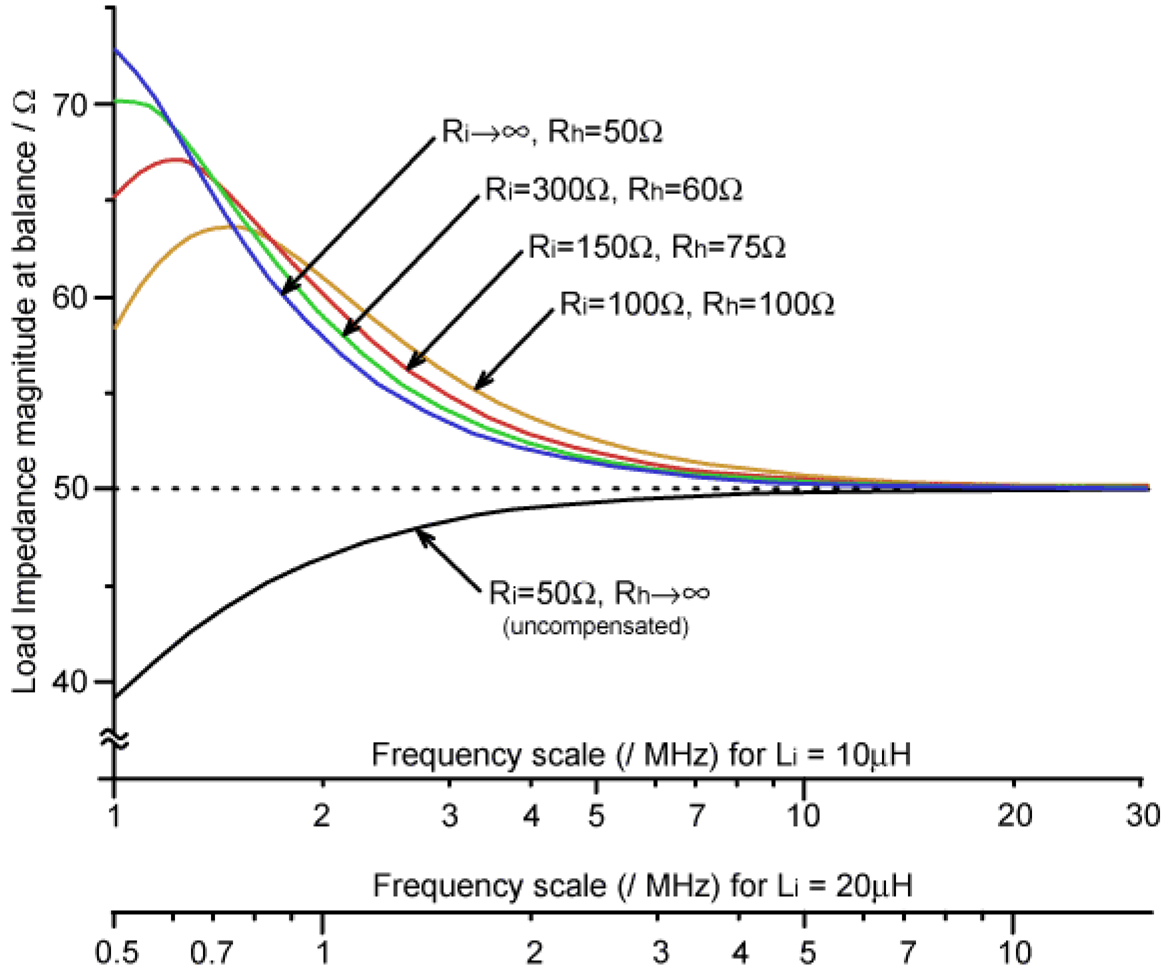

Fig. 4. Magnitude error of bridges with Herzog compensation and critical damping. Upper frequency scale is for $L_i = 10\ \mu H$, lower frequency scale is for $L_i = 20\ \mu H$.

	$R_i = 50\ \Omega$ $R_h \to \infty$ $C_h = 0$		$R_i = 100\ \Omega$ $R_h = 100\ \Omega$ $C_h = 1$ nF		$R_i = 150\ \Omega$ $R_h = 75\ \Omega$ $C_h = 1778$ pF		$R_i = 300\ \Omega$ $R_h = 60\ \Omega$ $C_h = 2778$ pF		$R_i = 1300\ \Omega$ $R_h = 52\ \Omega$ $C_h = 3698$ pF		$R_i \to \infty$ $R_h = 50\ \Omega$ $C_h = 4$ nF	
f / MHz	$\lvert Z_{bal}\rvert$	φ	$\lvert Z_{bal}\rvert$	φ	$\lvert Z_{bal}\rvert$	φ	$\lvert Z_{bal}\rvert$	φ	$\lvert Z_{bal}\rvert$	φ	$\lvert Z_{bal}\rvert$	φ
1.00	39.1	38.5	58.2	41.7	65.2	37.6	70.2	32.2	72.5	28.0	72.9	26.7
1.12	40.8	35.3	61.2	35.4	66.9	30.2	69.9	24.6	70.7	20.7	70.8	19.6
1.26	42.3	32.3	63.0	29.3	66.9	23.6	68.2	18.3	68.0	15.0	67.8	14.2
1.41	43.6	29.4	63.6	23.6	65.8	18.0	65.8	13.5	65.1	10.8	64.8	10.1
1.58	44.7	26.7	63.3	18.6	64.1	13.5	63.3	9.75	62.4	7.72	62.1	7.21
1.78	45.6	24.1	62.3	14.4	62.1	9.94	61.0	7.02	60.1	5.49	59.8	5.12
2.00	46.4	21.7	61.0	10.9	60.2	7.27	59.0	5.03	58.1	3.90	57.8	3.63
2.24	47.1	19.6	59.5	8.16	58.4	5.27	57.3	3.59	56.5	2.77	56.3	2.57
2.51	47.7	17.6	58.0	6.05	56.9	3.81	55.8	2.56	55.2	1.96	55.0	1.82
2.82	48.1	15.8	56.7	4.44	55.6	2.74	54.7	1.82	54.1	1.39	54.0	1.29
3.16	48.5	14.1	55.5	3.24	54.5	1.96	53.7	1.29	53.3	0.98	53.2	0.91
3.55	48.8	12.6	54.5	2.35	53.6	1.40	53.0	0.92	52.6	0.70	52.5	0.65
3.98	49.0	11.3	53.7	1.69	52.9	1.00	52.4	0.65	52.1	0.49	52.0	0.46
4.47	49.2	10.1	53.0	1.22	52,3	0.71	51.9	0.46	51.6	0.35	51.6	0.32
5.01	49.4	9.02	52.4	0.87	51.9	0.51	51.5	0.33	51.3	0.25	51.3	0.23
5.62	49.5	8.06	51.9	0.62	51.5	0.36	51.2	0.23	51.0	0.18	51.0	0.16
6.31	49.6	7.19	51.5	0.45	51.2	0.26	51.0	0.16	50.8	0.12	50.8	0.11
7.08	49.7	6.41	51.2	0.32	50.9	0.18	50.8	0.12	50.7	0.09	50.6	0.08
7.94	49.8	5.72	51.0	0.23	50.8	0.13	50.6	0.08	50.5	0.06	50.5	0.06
8.91	49.8	5.10	50.8	0.16	50.6	0.09	50.5	0.06	50.4	0.04	50.4	0.04
10.0	49.8	4.55	50.6	0.11	50.5	0.06	50.4	0.04	50.3	0.03	50.3	0.03
11.2	49.9	4.06	50.5	0.08	50.4	0.05	50.3	0.03	50.3	0.02	50.3	0.02
12.6	49.9	3.61	50.4	0.06	50.3	0.03	50.2	0.02	50.2	0.02	50.2	0.01
14.1	49.9	3.22	50.3	0.04	50.2	0.02	50.2	0.01	50.2	0.01	50.2	0.01
15.8	49.9	2.87	50.3	0.03	50.2	0.02	50.2	0.01	50.1	0.01	50.1	0.01
17.8	50.0	2.56	50.2	0.02	50.2	0.01	50.1	0.01	50.1	0.01	50.1	0.01
20.0	50.0	2.28	50.2	0.01	50.1	0.01	50.1	0.01	50.1	0.00	50.1	0.00
22.4	50.0	2.04	50.1	0.01	50.1	0.01	50.1	0.00	50.1	0.00	50.1	0.00
25.1	50.0	1.81	50.1	0.01	50.1	0.00	50.1	0.00	50.1	0.00	50.1	0.00
28.2	50.0	1.61	50.1	0.01	50.1	0.00	50.0	0.00	50.0	0.00	50.0	0.00
31.6	50.0	1.44	50.1	0.00	50.1	0.00	50.0	0.00	50.0	0.00	50.0	0.00

Table 2. Magnitude and phase error of Herzog LF-compensated bridges (data used for the preceding graphs, as obtained from the spreadsheet calculation **Herzog_LF.ods**).
$L_i = 10\ \mu H$, $R_0 = 50\ \Omega$, $R_i // R_h = 50\ \Omega$ (this model does not include propagation delay). The phases and magnitudes for any other transformer secondary inductance L_i' can be obtained by multiplying the entries in the frequency column by L_i / L_i', (where $L_i = 10\ \mu H$). The C_h values at the heads of the columns can be multiplied by L_i' / L_i to find the corresponding capacitance.

Discussion

When a current transformer bridge with no LF compensation is used to monitor the adjustment of an impedance matching network, the result is a transmitter load impedance that is too low in magnitude at low frequencies, a situation that is particularly damaging for transistor power-amplifiers[5]. The situation when Herzog's LF compensation is applied is an improvement, in that the load impedance magnitude obtained by balancing the bridge rises as the frequency is reduced (until the point at which the falling reactance of the transformer secondary coil begins to dominate); and although the bridge can not be considered to be accurate at low frequencies, the error incurred is harmless in that it will merely cause a small reduction in transmitter output power. Loading a transmitter with 60 Ω instead of 50 Ω will not be noticeable to the operator, and will have little effect in terms of signal strength at the receiving station; and so we might regard a magnitude error of +20% as acceptable. This implies an SWR of 60/50 = 1.2 on any 50 Ω cable between the transmitter and the bridge, which is not serious. The point with which we might take issue however, is that the resulting bridge is inaccurate, whereas bridges compensated by modification of the voltage sampling network can be engineered for near-perfect performance[6].

With regard to the phase performance (refer to **fig. 3**), we might observe that critical damping of the current transformer secondary network is very effective at bringing the load phase angle close to 0° over a wide frequency range. If we adopt the criterion recommended by Underhill and Lewis[7], that a phase error of 7° or less is inoffensive to the generator, we see that an uncompensated bridge with a 10 μH transformer secondary and a 50 Ω secondary load is unsatisfactory at frequencies below about 7 MHz, whereas the maximally-boosted Herzog-compensated bridge (R_h = 50 Ω , R_i omitted) gives acceptable performance down to about 1.7 MHz. We can also note that the maximally-boosted circuit gives the best compensation overall; an outcome that we might have expected given that R_i does not appear in the condition for critical damping (equation **2**). It is probable that the reason why Herzog used partial LF-boost (finite values for both R_i and R_h) in his patent is that he needed R_i to be present so that he could place a small inductor in series with it in order to compensate for the propagation delay or parasitic parallel secondary capacitance of the current transformer. In fact, he could have placed such an inductance in series with R_h without significantly affecting the LF compensation, but he perhaps did not realise this. Herzog's HF phase compensation method is analysed and tested experimentally in a separate article[8].

The principal advantage of Herzog's LF compensation scheme is that it can be used when modification of voltage sampling network is either difficult or undesirable; and results in a bridge that, although inaccurate, behaves in a way that will not reduce the life-expectancy of any transistor power amplifier connected to it. Given that it is already possible to make accurate bridges by adjusting the voltage sampling network to have the same frequency dependence as the current sampling network, it appears to have little merit it in its intended form; but as is so often the case, what the inventor had in mind is not the reason why the circuit is interesting. Since no theoretical analysis was given in the original references, it is difficult to know how thoroughly the circuit had been investigated when the patent was written; but the simple view that the series capacitor reduces the loading on the current transformer at low frequencies does not do justice the actual circuit behaviour. When the compensating capacitor is chosen to give best phase accuracy, there is a substantial increase in current transformer output voltage as the frequency is reduced, which suggests that we might connect to the network in a different way in order to counteract this

5 See discussion of power-transfer efficiency given in **AC Theory** (previously cited) **section 34**.

6 **Evaluation and optimisation of RF current-transformer bridges**. D W Knight. http://g3ynh.info/zdocs/bridges/ .

7 **Automatic Tuning of Antennae**. M J Underhill (G3LHZ) and P A Lewis, SERT Journal, Vol 8, Sept 1974, p183-184. Reprint of paper in Mullard Research Labs Annual Review, 1973. Gives criteria for achieving 1.2:1 SWR, i.e., $45 \le R \le 56$ Ω , $17.5 \le G \le 22.5$ mS , $-7° \le \varphi \le +7°$.

8 **Evaluation and optimisation of RF current-transformer bridges** (cited above). **Sections 13** and **18e**.

tendency. This leads to a circuit rearrangement (not covered by Herzog's patent) that is useful for the construction of precision RF ammeters, and is referred to by this author as the "***maximally-flat current-transformer***" (**fig. 5**). A maximally-flat amplitude response (which is what matters for scalar ammeters) can be obtained by placing a suitably-chosen capacitor in series with the secondary load resistor and measuring the magnitude of the voltage across the resistor (rather than across the secondary coil) [9, 10]. The phase performance of the resulting network is however inferior to that of the conventional current-transformer.

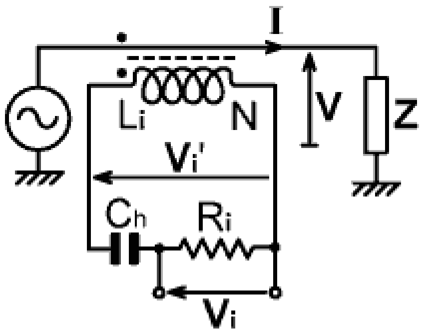

Fig. 5.
Maximally-flat current transformer network.

Condition for maximal amplitude flatness is obtained when:
$R_i^2 - 2L_i/C_h = 0$

DWK 2007, 2014.

9 **The maximally-flat current transformer**, D W Knight, www.g3ynh.info/zdocs/bridges .
10 **Amplitude response of conventional and maximally-flat current transformers**, D W Knight, www.g3ynh.info/zdocs/bridges .